# PARAMETRIC IDENTIFICATION OF A NON-LINEAR MODEL OF A MARINER CLASS VESSEL

E. Moyano (1), E. López (2), T. M. Rueda (3), F. J. Velasco (3)

- (1) Dpto. Matemática Aplicada y C. de la Computación. U. Cantabria (Spain)
- (2) Dpto. Ciencias y Técnicas de la Navegación, Máquinas y Construcciones Navales. UPV/EHU (Spain)
- (3) Dpto. Tecnología Electrónica, Ing. de Sistemas y Automática. U. Cantabria (Spain)

## ABSTRACT

This paper describes a discrete linear model for the course-changing manoeuvres of a ship. A non-linear mathematical model of three degrees of freedom is used. The linear model has been obtained from a parametric model. The coefficients are obtained by means of identification. In order to validate the linear model, a comparison has been made of the responses of the non-linear model and the identified model for various course-changing manoeuvres.

## **KEYWORDS**

Ship control, autopilots, marine systems, control systems, ship model, course-changing control, system identification.

#### **INTRODUCTION**

Mathematical models of ship steering dynamics are useful for computer simulations and for autopilot design. The parameter values of such models are normally estimated from theoretical calculations or from scale model tests [1].

The requirements on ship steering are increasing for reasons of safety and economics. PDI-control algorithms have been widely used in the autopilots. These controllers require adjustments to compensate for various seakeeping circumstances. Fixed settings are therefore often used and thus the autopilot does not work in optimum conditions.

In order to overcome these disadvantages and to obtain a good operating level for all seakeeping conditions, robust or adaptive control techniques have been used recently in the design of these controllers [2].

The problem with robust feedback control system design is to synthesize a control law which maintains system response to within prespecified tolerances despite the effects of uncertaity on the system. For a ship, this uncertainty can take the form of seawaves, winds, currents, etc.

For some cases of robust controller design such as  $H_{\infty}$  or  $\ell_1$  the system must be described using its transfer function. The transfer function of a ship can be obtained from a non-linear model of its movement, with three degrees of freedom. This is possible thanks to the system identification theory, which makes it possible to find a discrete linear model which can be used to simulate robust ship steering control strategies.

# MATHEMATICAL MODEL OF THE VESSEL

Six independent coordinates are required to represent the movement of a ship. Three coordinates are used to describe the translation movements around the axes  $x_B$ ,  $y_B$  and  $z_B$  (surge, sway and heave respectively), referring to a system of mobile coordinates situated in the ship  $O_B$ . Another three coordinates describe the rotation movements (roll, pitch and yaw respectively) of the mobile coordinates system of the ship with respect to the inertial reference coordinates system situated on land O.

For the Mariner ship [3] contemplated in this study, the movements of heave, roll and pitch can be considered null due to their low values in comparison with other movements. Thus the mathematical model of the ship is considered in surge, sway and yaw motions and the movement of the ship can be represented using Newton equations with three degrees of freedom, by the following equations:

surge: 
$$m(\dot{u} - vr - x_G r^2) = X$$
  
sway:  $m(\dot{v} - ur + x_G \dot{r}) = Y$  (1)  
yaw:  $I_{zz}\dot{r} + m x_G (ur + \dot{v}) = N$ 

where *m* is the mass of the ship and the centre of gravity is assumed to be in the position  $(x_G, 0, 0)$   $(x_G$  is the distance at axis  $x_B$  to gc).  $I_{ZZ}$  is the inertial moment of the ship about  $z_B$  axis, *u* is the linear surge and *v* the linear sway velocities. The angular velocity, yaw, is represented by *r* and the yaw angle is  $\psi$  measured in the inertial frame  $(\dot{\psi} = r)$ .

The terms X and Y denote the hydrodynamic forces acting along the axes  $x_B$  and  $y_B$ . N is the hydrodynamic moment around the  $z_B$  axis. These quantities take into account the hydrodynamic effects of the movements of the hull, the forces and moments exercised on the ship by the propeller and the rudder and the influence of the wind, waves and currents.

There is a wide range of approaches to developing a set of non-linear equations for movement, differing basically in their expression of the hydrodynamic forces X and Y and the hydrodynamic moment N of eq. (2) which are functions of the ship's movement. In [4] [5] the use of a third order truncated Taylor series of the X, Y and N functions at  $u = u_0$  and  $v = r = p = \dot{u} = \dot{v} = \dot{r} = \dot{p} = 0$  has been proposed in the equation

$$[X Y N]^{T} = f(u, v, r, \dot{u}, \dot{v}, \dot{r}, \delta)$$
<sup>(2)</sup>

In the series development proposed, no terms higher than the third order are included since experience has shown that their inclusion does not significantly increase accuracy. In [6], a detailed presentation is made of the Mariner ship model used in this paper. As well as the ship's hydrodynamics, the saturations in the rudder mechanics have also been included. To include the rudder action in the model, the simplified model proposed by [7] has been used.

#### MODEL IMPLEMENTATION

The non-linear model resulting from the above considerations has been implemented in the Matlab\_Simulink environment using an S-function. A trapezoidal signal generator has been designed which enables the selection of rudder rotation speed, the time the rudder remains at a constant angle, and frequency of rudder oscillation. The signals can have constant parameters or random variation within a limited range of variation. The rudder angle and rudder rate limiters will typically be in the ranges:  $\delta_{\text{max}} = \pm 35^{\circ}$ , and  $2.5^{\circ}/\text{sec} \le \dot{\delta}_{\text{max}} < 7^{\circ}/\text{sec}$  respectively.

#### **IDENTIFICATION-VALIDATION**

Assuming unit sampling interval, there is one input signal u(t), t = 1...N and one output signal y(t), t = 1...N. Assuming the signals are related by a linear system, the relationship can be written

$$y(t) = G(q) u(t) + v(t)$$
 (3)

where q is the shift operator and G(q) u(t) is short for

$$G(q)u(t) = \sum_{k=1}^{\infty} g(k) u(t-k)$$
 (4)

The function G(q) is called the transfer function of the system.

A transfer function can be described as a rational function of  $q^{-1}$  ( $q = e^{i\omega}$ ) where the numerator and denominator coefficients can be specified in some way. A commonly used parametric model is the ARX model [8] that corresponds to

$$G(q) = q^{-nk} \frac{B(q)}{A(q)}$$
(5)

where B y A are polynomials in the delay operator  $q^{-1}$ .

$$A(q) = 1 + a_1 q^{-1} + \dots + a_{na} q^{-na}$$
  

$$B(q) = 1 + b_1 q^{-1} + \dots + b_{nb} q^{-nb}$$
(6)

na and nb are the orders of the respective polynomials. The number nk is the number of delays from input to output.

Henceforth, these models shall be referred to, with the notation (na, nb, nk). With the model description and having observed the input-output data u and y, the prediction errors e(t) can be computed as

$$e(t) = y(t) - G(q) u(t)$$
(7)

These errors are, for given data y and u, functions of G. The parametric identification method estimates G minimising  $V_N(G) = \sum_{t=1}^N e^2(t)$ . This is called a prediction error method.

A model structure common to all the states under study is desired. In order to select the model order, Akaike's final prediction error (FPE) criteria has been used and possible cancellations of poles and zeros have been considered.

#### **Identification procedure**

As input, three types of series of movements of the rudder have been considered: random variation in the opening angle with constant frequency; random variation in the opening angle with varying frequency; and zig-zag movement. In all cases, a duration of 3.000 seconds has been considered to improve the reliability of the identification. A rudder rotation rate of 6°/sec has been chosen.

For each of these signals, the corresponding output has been obtained from the nonlinear model. Input-output pairs are used to obtain the model by means of identification. The objective is to find a common structure for the different inputs which, if possible, is the one with the best adjustment in all three cases. The signals used as input are shown below in figures 1, 2 and 3.





Some of the structures identified are shown below with their corresponding errors, for each of the input signals in tables 1, 2 and 3.

Table 1.-Regular signals

| STRUCTURE  | FPE          |
|------------|--------------|
| (na,nb,nk) |              |
| (3, 2, 0)  | 2.85773e007  |
| (4, 1, 0)  | 4.12372e-008 |
| (4, 1, 1)  | 4.95769e-008 |
| (5, 1, 0)  | 2.96169e-008 |
| (5,1,1)    | 3.51791e-008 |
| (5, 2, 0)  | 2.5812e-008  |

Table 2.-Variable signals

| STRUCTURE  | FPE          |
|------------|--------------|
| (na,nb,nk) |              |
| (3, 2, 0)  | 4.17358e-008 |
| (4, 1, 0)  | 1.36784e-008 |
| (4, 1, 1)  | 1.40648e-008 |
| (5, 1, 0)  | 1.31305e-008 |
| (5,1,1)    | 1.34644e-008 |
| (5, 2, 0)  | 1.22748e-008 |

## Table 3.-Zig-zag signal

| STRUCTURE  | FPE          |
|------------|--------------|
| (na,nb,nk) |              |
| (3, 2, 0)  | 9.93911e-007 |
| (4, 1, 0)  | 3.02449e-007 |
| (4, 1, 1)  | 1.98271e-007 |
| (5, 1, 0)  | 1.95122e-007 |
| (5,1,1)    | 1.41278e-007 |
| (5, 2, 0)  | 1.35848e-007 |

It can be observed in all cases that systems with more than two zeros do not improve the adjustment. Similarly, the introduction of delays, while it does not change the FPE much, does change the trajectory. Systems with a greater number of poles are not considered, since these increase complexity without improving the adjustment.

In the three situations contemplated, it can be observed that the structure (5, 1, 0) minimises the FPE. The polynomials A(q) and B(q) in this case are:

$$A(q) = 1 - 4.08 \ q^{-1} + 6.768 \ q^{-2} - 5.724 \ q^{-3} + 2.466 \ q^{-4} - 0.4301 \ q^{-5}$$

$$B(q) = 0.0001311$$

## Validation

With the above discrete model and with the same input signals, the signals for the validation of the model are obtained as output. The figures below show the comparison of the output signals of the initial model with those of the identified model.



Figure 6.- Zig-zag signal adjustment.



## CONCLUSIONS

In this paper, the transfer function of a discrete linear model of ship steering is obtained from a non-linear mathematical model. The mathematical model has been implemented in the Matlab-Simulink environment by means of an S-function.

The transfer function has been obtained by means of parametric identification. This discrete linear model has been validated with various course-change manoeuvres, obtaining in all cases a good agreement with the data from the original model.

# ACKNOWLEDGEMENTS

CICYT of Spain supported this development under contracts TAP97-0607-C03-03, DPI2000-0386-C03-03 and DPI2003-09745-C04-03.

## REFERENCES

- [1] FOSSEN, T. I. *Marine Control Systems*. Marine Cybernetics. 2002.
- [2] KALLSTROM, C. G.: *Identification and Adaptive Control Applied to Ship Steering*. Tesis Doctoral, *Lund Institute of Technology* Department of Automatic Control, Sweden. 1979.
- [3] CHISELETT M. S., STROM-TEJSEN J.: Planar Motion Mechanism Tests and fullscale Steering and Maneuvering Predictions for a Mariner Class Vessel, *Technical Report Hy-6* hydrodynamics Department, hydro- and Aerodynamics Laboratory, Lyngby, Denmark. 1965.

- [4] ABKOWITZ M. A. Lectures on Ship Hydrodynamics, Steering and Manoeuvrability. *Technical Report Hy-5* Hydrodynamics Department, hydro- and Aerodynamics Laboratory, Lyngby, Denmark. 1964
- [5] LEWIS, E.V. *Principles of Naval Architecture (2<sup>a</sup> ed.)*, Vol. 3. The Society of Naval Architects and Marine Engineers. 1989.
- [6] LÓPEZ, E., VELASCO, F. J., RUEDA, T. M. Modelos matemáticos de Buques . I Congreso Nacional de Investigación e Innovaciones Tecnológicas en el Ámbito Marítimo. Barcelona. 1997. 671-680
- [7] AMERONGEN J. van, Adaptive steering of ships a model reference approach to improved manoeuvring and economical course keeping, Ph.D. thesis, Delft University of Technology, 1982
- [8] LJUNG, L. System Identification: Theory for the user. Prentice Hall. USA. 1999