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Abstract: Experiments in towing tanks are concerned with the determination of the motion transfer functions, which are obtained by testing in irregular waves. For this purpose a model of pitch and heave motions has been developed, and the identified models of the vertical plane motions of a high-speed craft are showed. Linear models are obtained for different sea states and ship speeds. A general low order model is obtained by pole-zero cancellation. So, a full model may be considered for simulation and verification, and a low order model for design. From the lowest order models, a state space model is obtained.

Keywords: identification, autoregressive models, state space realization, ship modelling.

1. INTRODUCTION

The main problem for the development of a high-speed craft is concerned with the passenger's comfort and the safety of the vehicles. The vertical acceleration associated with roll, pitch and heave motions is the cause of motion sickness. The roll control is the most attractive candidate for control since increasing roll damping can be obtained more easily. However, shipbuilders are also interested in increasing pitch and heave damping. In order to solve the problem, anti-pitching devices and pitch control methods must be considered. Previously, models for the vertical ship dynamic must be developed for the design, evaluation and verification of the results.

The number of published investigations about ship modelling is immense. For example, a nonlinear model in 6 degree of freedom is shown in Fossen and Fjellstad (1995), a survey of ship models and experimental techniques for identification of ship dynamics are described in several publications (see for example: Fossen, 1994; Webster, 1992; Lloyd, 1989).

The hydrodynamic and derivatives coefficients occurring in the equations of motion cannot be calculated analytically and hence tests with the physical model are carried out in towing tanks, rotating arms tanks and Planar Motion Mechanism (PMM). Experimental techniques (as described in Lewis, 1989, or Linkes, 1980) can be used to determine these coefficients.

The system identification techniques (Ljung, 1987) have also been utilised to develop hydrodynamic coefficients for mathematical modelling of ship trajectories (see for example the early work of Aström and Kälström, 1976). Another alternative is to apply a state augmented extended Kalman filter (Gelb, 1986) to estimate the ship parameters (see for example Fossen *et al.*, 1996). Usually, different manoeuvres and PRBS input signals (*via* steering machine or helmsman) are defined for identification.

Experiments in towing tanks are concerned with the determination of the motion transfer functions. Usually, tests in regular waves are made to experimental determination of the motion transfer functions. In this case, it is necessary to record the sinusoidal motions of the model and to determine the motion amplitudes experienced for a variety of different waves frequencies. The incident waves can be measured using a wave probe mounted on the towing carriage. This introduces a phase shift in the recorded motions and it is necessary to correct for this effect in the analysis.

Also, the transfer function can be obtained by testing in irregular waves. In this case the phase for each frequency component should be corrected, but this is not possible with a continuous spectrum. Instead a rational approximation is used.

The aim of this paper is to develop a model of pitch and heave motions. The model identification of vertical plane motions (pitch and heave) of a high speed craft (Turbo Ferry TF-130) by testing it using irregular waves in the towing tank of the CEHIPAR is showed. A high order model is identified and later a lower order model with a delay is proposed.

The model is obtained by system identification techniques following the three usual major steps: a) model structure determination, b) parameter estimation, and c) model validation.

2. LINEAR EQUATIONS OF VERTICAL MOTION

As noted in Lewis (1989), the separation of the response of the ship into vertical and horizontal plane motions is the result of linear theory and the fact that for a port/starboard symmetric ship there is no cross-coupling between them.

The linear equations of pitch and heave motions are (see Lewis, 1989):

$$\begin{aligned}&\Delta\ddot{h} - \Delta x_G \ddot{\theta} = \\ &-\left[A_{33}\ddot{h} + B_{33}\dot{h} + C_{33}h + A_{35}\ddot{\theta} + B_{35}\dot{\theta} + C_{25}\theta\right] + F_{ex_z}\end{aligned} \tag{1}$$

$$\begin{aligned}&I_{yy}\ddot{\theta} - \Delta x_G \ddot{h} = \\ &-\left[A_{53}\ddot{h} + B_{53}\dot{h} + C_{53}h + A_{55}\ddot{\theta} + B_{55}\dot{\theta} + C_{55}\theta\right] + M_{ex}\end{aligned} \tag{2}$$

where:

Δ	is the total mass of vessel,
h	is the response of ship to waves in heave mode,
$(x_G, 0, z_G)$	are co-ordinates of the centre of gravity of the ship,
θ	response of ship to waves in pitch mode,
A_{jk}	overall added mass,
B_{jk}	overall damping,
C_{jk}	overall restoring, (subscripts j,k indicate modes)
I_{yy}	inertia moment around the y-axis,
F_{ex_z}	Exciting force due to waves,
M_{ex}	Exciting moment due to waves.

From here, one can obtain:

$$\begin{aligned}&(\Delta + A_{33})\ddot{h} + (A_{35} - \Delta x_G)\ddot{\theta} = \\ &= -\left[B_{33}\dot{h} + C_{33}h + B_{35}\dot{\theta} + C_{35}\theta\right] + F_{ex_z}\end{aligned} \tag{3}$$

$$\begin{aligned}&(A_{53} - \Delta x_G)\ddot{h} + (I_{yy} + A_{55})\ddot{\theta} = \\ &= -\left[B_{53}\dot{h} + C_{53}h + B_{55}\dot{\theta} + C_{55}\theta\right] + M_{ex}\end{aligned} \tag{4}$$

By Laplace transform and with a second order approximation for forces and moments:

$$\begin{aligned}&\left[(\Delta + A_{33})s^2 + B_{33}s + C_{33}\right]h + \\ &+ \left[(A_{35} - \Delta x_G)s^2 + B_{35}s + C_{35}\right]\theta = \\ &= \left(f_1 s^2 + f_2 s + f_3\right)u\end{aligned} \tag{5}$$

$$\begin{aligned}&\left[(A_{53} - \Delta x_G)s^2 + B_{53}s + C_{53}\right]h + \\ &+ \left[(I_{yy} + A_{55})s^2 + B_{55}s + C_{55}\right]\theta = \\ &= \left(m_1 s^2 + m_2 s + m_3\right)u\end{aligned} \tag{6}$$

where u is the wave surface elevation.

From here it is easy to see that:

$$h(s) = \frac{H_{22}(s)F(s) - H_{12}(s)M(s)}{H_{11}(s)H_{22}(s) - H_{21}(s)H_{12}(s)}u(s) \tag{7}$$

$$\theta(s) = \frac{H_{11}(s)M(s) - H_{21}(s)F(s)}{H_{11}(s)H_{22}(s) - H_{21}(s)H_{12}(s)}u(s) \tag{8}$$

where:

$$H_{11}(s) = (\Delta + A_{35})s^2 + B_{33}s + C_{33}$$
$$H_{12}(s) = (A_{35} - \Delta x_G)s^2 + B_{35}s + C_{35}$$
$$H_{21}(s) = (A_{53} - \Delta x_G)s^2 + B_{53}s + C_{53}$$
$$H_{22}(s) = (I_{yy} + A_{55})s^2 + B_{55}s + C_{55}$$
$$F(s) = f_1 s^2 + f_2 s + f_3$$
$$M(s) = m_1 s^2 + m_2 s + m_3$$

The model obtained by identification is discrete, so a discretization of (7) and (8) must be made.

The discrete transfer functions corresponding to (7) and (8) can be written as:

$$\begin{pmatrix} h(z) \\ \theta(z) \end{pmatrix} = G(z)u(z)$$
$$G(z) = \frac{1}{d(z)} \cdot \left[N_1\, z^{r-1} + ... + N_r \right] \tag{9}$$
$$d(z) = z^r + d_1\, z^{r-1} + ... + d_r$$

where, $d(z)$ is the least common multiple of the denominators (of r order) and N_i are matrices with the relevant coefficients of the numerators.

A representation in state space can be obtained by a canonical realization (see for example Kailath, 1980). So, the following is a realization of (9):

$$\begin{aligned} x(k+1) &= A\, x(k) + B\, u(k) \\ y(k) &= C\, x(k) \end{aligned} \tag{10}$$

where:

$$A = \begin{bmatrix} -d_1 & -d_2 & . & . & -d_r \\ 1 & 0 & . & . & 0 \\ 0 & 1 & . & . & 0 \\ 0 & 0 & . & . & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}_{rxr} \quad B = \begin{bmatrix} 1 \\ 0 \\ . \\ . \\ 0 \end{bmatrix}_{rx1} \quad C = \begin{bmatrix} n_{11} & n_{12} & . & . & n_{1r} \\ n_{21} & n_{22} & . & . & n_{2r} \end{bmatrix}_{2xr} \tag{11}$$

3. SCALE-MODEL TESTING

A model test was carried out in the towing tank of the CEHIPAR. The model was free to move in heave direction and pitch angle. In the experiments, the sea states (SSN), according to STANAG 4194 (Standarized Wave and Wind Environments and Shipboard Reporting of Sea Conditions), were 4, 5 and 6.

The waves spectrum was JONSWAP type (see Figure 1). The wave surface elevation was measured at 3.8 m. forward from model bow (this is equivalent to 95 m. in full scale).

This introduces a phase lead in the recorded motions for each wave frequency component. The phase lead for each wave frequency is kx, where k is the wave number ($k=\omega^2/g$), and x is the distance from the bow to the wave probe, ω is the wave component frequency, g is the gravity. The phase lead is proportional to ω^2. A rational approximation for this effect can be made and its zero-pole distribution is similar to that shown in Figure 2.

Furthermore, different ship speeds and courses were tested. In the first and second columns of Tables 1 and 2 are showed the sea states and ship speeds considered for testing.

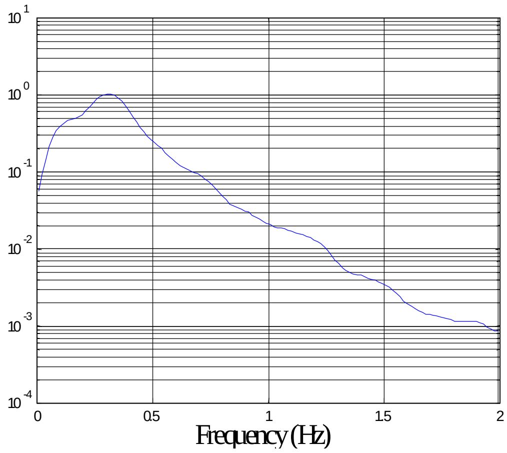

Fig. 1: Wave spectrum for test with a speed of 30 knots and sea state 5.

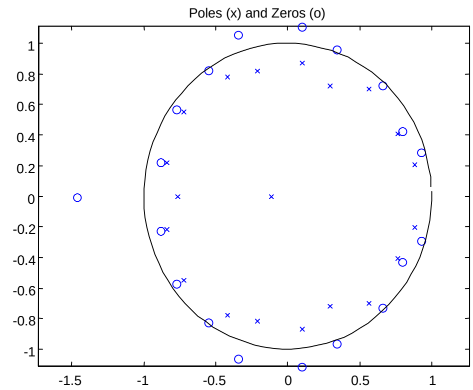

Fig. 2: Zeros-poles distribution for the waves phase lead.

4. IDENTIFICATION PROCEDURE

In this section the pitch and heave dynamics are identified according to the mathematical model proposed in Section 2. In these models the phase lead approximation of section 3 is considered. Hence, ARX models are tested:

$$A(q)y(t) = B(q)u(t - n_k) + e(t) \tag{12}$$

where n_k is the time delay, A and B are polynomials in the delay operator of orders n_a and n_b, respectively. The sampling rate is 4 Hz.

The model structure has been selected by Akaike's criterion, and possible pole-zero cancellation has been considered. Tables 1 and 2 show, respectively, the orders of the best models for pitch and heave dynamics and the final prediction error for every sea state and ship speed. With this criterion a different structure for each condition has been obtained.

A common structure for all the models can be considered. This structure is chosen by an iterative procedure, the different structures are compared and the structure with less FPE value for all conditions is selected.

Table 1: Model structure for pitch dynamic

Sea state	Ship speed (knots)	Orders of the best ARX models (n_a,n_b,n_k)	FPE*10^{-3} (degrees(°)^2) of the best ARX models	FPE*10^{-3} ((°)^2) ARX model (10,10,10)
4	20	(10,10,8)	3.0893	3.1365
	30	(9,10,10)	8.8617	9.1222
	40	(9,10,9)	18.045	18.792
5	20	(10,10,10)	1.5895	1.5895
	30	(2,10,10)	21.913	42.906
	40	(9,9,2)	10.430	10.739
6	20	(8,10,8)	1.8767	1.8968
	30	(10,8,10)	4.8242	4.9255
	40	(10,10,6)	7.5757	8.3722

Table 2: Model structure for heave dynamic

Sea state	Ship speed (knots)	Orders of the best ARX models (n_a,n_b,n_k)	FPE*10^{-4} (m^2) of the best ARX models	FPE*10^{-4} (m^2) ARX model (8,7,5)
4	20	(10,5,6)	3.5028	4.1930
	30	(3,7,7)	4.169	4.4144
	40	(7,8,6)	5.1802	5.4957
5	20	(8,10,4)	4.6171	4.8438
	30	(10,9,3)	7.5599	7.8218
	40	(8,10,4)	10.970	11.218
6	20	(10,10,2)	5.7798	5.8848
	30	(8,6,6)	9.9269	9.8687
	40	(8,7,5)	21.769	21.190

Table 3: FPE of low order models

Sea state	Ship speed (knots)	FPE*10^{-3} ((°)^2) pitch model (4,4,10)	FPE*10^{-4} (m^2) heave model (4,3,5)
4	20	8.3695	4.7912
	30	22.648	5.5650
	40	32.925	6.1778
5	20	3.5991	6.3880
	30	42.626	9.2055
	40	16.058	13.512
6	20	3.8712	6.8929
	30	12.431	11.914
	40	20.236	25.612

The common structure is (10,10,10) for pitch dynamic and (8,7,5) for heave dynamic. The last column of Tables 1 and 2 show the FPE of the models with this structure for every sea state and ship speed.

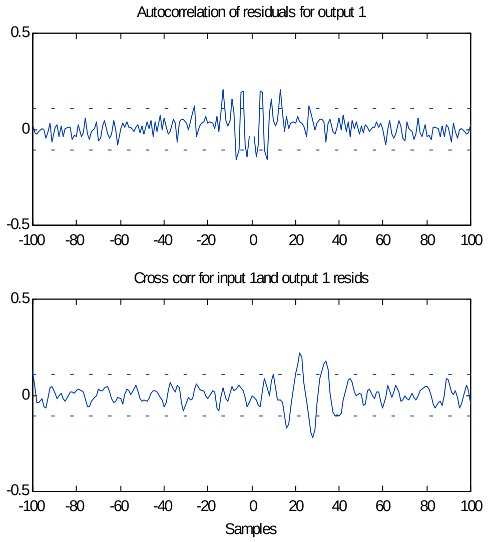

Fig. 3: Autocorrelation of residuals of the low order pitch dynamic model for sea state 5 and a ship speed 30 knots.

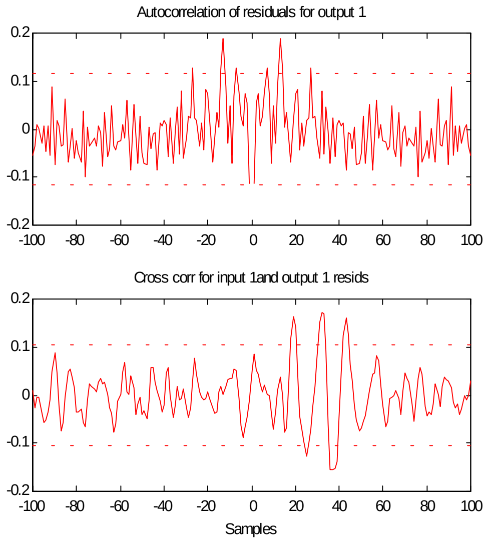

Fig. 4: Autocorrelation of residuals of the low order heave dynamic model for sea state 5 and a ship speed 30 knots.

As can be seen, for every condition the models with the common structure have values of FPE that are slightly worse than the FPE values obtained with the best ARX models. Nevertheless, the difference is small and the common structure can be considered for all conditions of sea state and ship speed.

Then, a low order model has been obtained by pole-zero cancellation, and the phase lead for each frequency component has been approximated by a simple delay.

Table 3 shows the FPE values for these models. Figures 3 and 4 show the autocorrelation of the residuals of the pitch and the heave models for sea state 5 and a ship speed of 30 knots. As can be seen from this example and for validation purpose, the residuals of the low order models are acceptable.

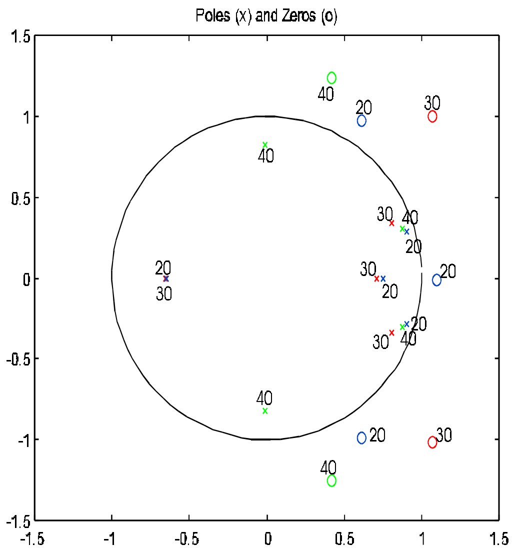

Fig. 5: Poles (x) and zeros (o) of the pitch low order model in function of ship speed.

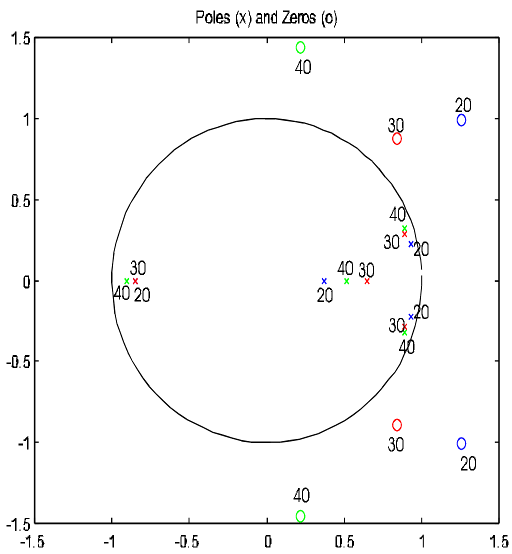

Fig. 6: Poles (x) and zeros (o) of the heave low order model in function of ship speed.

Therefore, a full model is considered for simulation and verification, and a low order one for design, with parameters that are function of ship speed and sea state. Figures 5 and 6 show the poles and zeros of the pitch and heave low order models in function of ship speed. The ship speeds are marked with numbers 20, 30 and 40 close to the relevant poles and zeros; the sea state is 5. Figure 7 shows the pitch Bode plot in function of ship speed (the solid line corresponds with ship speed of 20 knots, the dashed line with 30 knots and the dotted line with 40 knots).

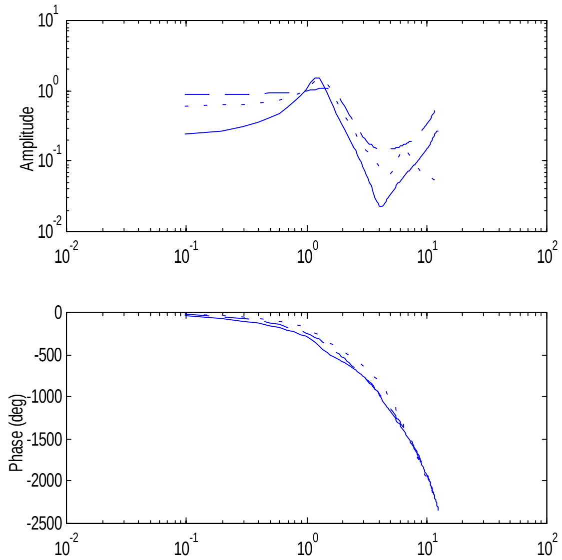

Fig. 7: Bode plot of the pitch low order model.

A similar behaviour is observed with sea state conditions, figures 8 and 9 show the poles and zeros in function of sea state. (The sea states are marked with numbers 4, 5 and 6 close to the relevant poles and zeros; the ship speed is 40 knots).

From these lowest order models, a representation in state variables is obtained according to Section 2. In this model the coupling between pitch and heave motions can be seen. For example, with sea state 5 and a ship speed of 40 knots the transfer functions are:

$$h(z) = \frac{0.0113z^2 - 0.0048z + 0.0243}{z^7 - 1.403z^6 - 0.253z^5 + 1.181z^4 - 0.421z^3} u(z)$$

$$\theta(z) = \frac{-0.0158z^3 + 0.08894z^2 - 0.0896z + 0.1301}{z^{13} - 1.737z^{12} + 1.515z^{11} - 1.182z^{10} + 0.5874z^9} u(z)$$

The dominant poles in the heave model correspond to a complex conjugate pair with a damping of 0.145 and a natural frequency of 1.381 rad./sec. In the pitch mode the dominant poles correspond to a complex conjugate pair with a damping of 0.222 and a natural frequency of 1.383 rad./sec.

The state space representation for this case is given by (11), where the coefficients d_i of matrix A and n_{ij} of matrix C are showed in Table 4.

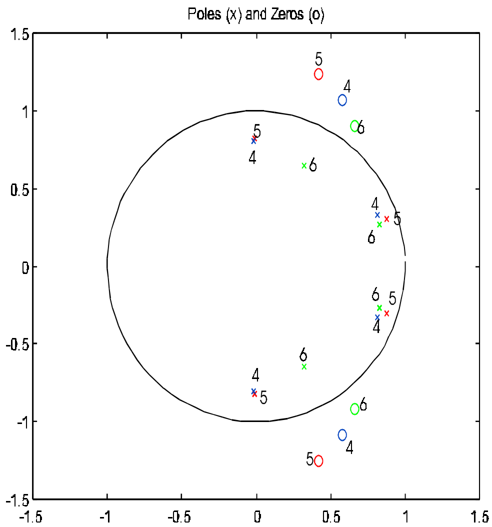

Fig. 8: Poles (x) and zeros (o) of the pitch low order model in function of sea state.

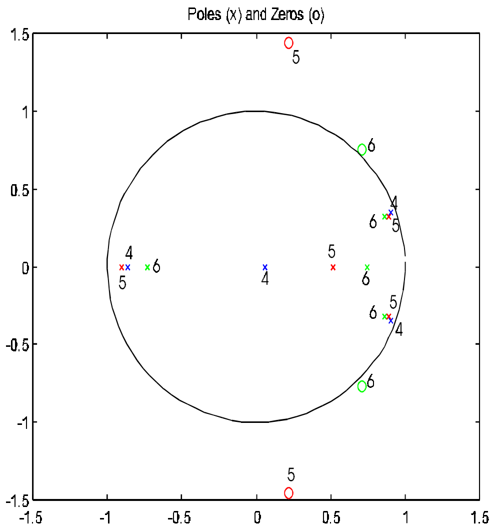

Fig. 9: Poles (x) and zeros (o) of the heave low order model in function sea state.

Table 4: Coefficients of matrices A and C

j	$-d_j$	n_{1j}	n_{2j}
1	3.1401	0	0
2	-3.6997	0	0
3	1.6880	0	0
4	0.6094	0	0
5	-1.9951	0.0113	0
6	2.1820	-0.0245	0
7	-1.1911	0.0499	0
8	0.2472	-0.0630	0
9	0	0.0493	0
10	0	-0.0316	-0.0158
11	0	0.0143	0.1111
12	0	0	-0.2104
13	0	0	0.2147
14	0	0	-0.0482
15	0	0	-0.1762
16	0	0	0.1914
17	0	0	-0.0548

5. CONCLUSIONS

System identification techniques have been used in order to obtain a mathematical model for the vertical dynamics of a high speed craft. A collection of tests was performed using irregular waves in a towing tank. The data collected have been used to adjust the parameters of different low order ARX models for the pitch and heave dynamics.

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