GARTEUR GROUP FOR AERONAUTICAL RESEARCH AND TECHNOLOGY IN EUROPE

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GARTEUR Open

# ROBUST CONTROL TECHNIQUES
# TUTORIAL DOCUMENT

by

FM(AG08)

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This report has been prepared under auspices of
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and Integration of the Group for Aeronautical
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## Summary

In GARTEUR Action Group FM(AG08) a design challenge is performed in order to demonstrate how robust flight control theory can be applied to realistic problems and also to demonstrate the benefits and limitations of such techniques. For this purpose two benchmark problems have been defined: the Research Civil Aircraft Model (RCAM) and the High Incidence Research Model (HIRM). Twenty-two teams participate in the challenge, together applying the following methods:

1. Robust Eigenvalue/Eigenstructure Assignment
2. Non-Linear Dynamic Inversion (NLDI)
3. Lyapunov Methods
4. Space Parameter
5. $H_\infty$
6. Linear Quadratic (LQ) & Associated Techniques (LQG/LTR)
7. Predictive Control
8. Robust Inverse Dynamic Estimation (RIDE)
9. Structured Singular Values. μ Analysis & Synthesis
10. Multi Objective Parameter Tuning
11. Fuzzy Logic
12. Classical control

A short description of each of these techniques is given, except for the classical control. The main objective of this report is to support a comparative study about applicability, advantages and drawbacks of the methods. In face of this activity, which has been performed during the Evaluation Phase of the project, each technique is treated according to the same structure with sections on the global goals of the method, the systems model description, the controller structure, the design specifications, the analysis approach, the controller synthesis approach, practical implementation aspects and the relation with other methods. Another objective is to provide the committees that will review the HIRM and RCAM design documents with information about the techniques that are being applied.

Two techniques which are not applied by the design teams are treated as well:

13. Quantitative Feedback Theory
14. Kharitonov Theory

Finally, some theoretical background about Lyapunov concepts, Singular Values, Vector, Signal and System Norms and Linear Matrix Inequalities are included in a Mathematical Appendix. Also, the basic concept for the MIMO system analysis are described in the Control Theory Background appendix.

# 4 Non Linear Dynamic Inversion Technique

## 4.1 Theoretical aspects

### 4.1.1 Global goal

Over the last two decades or so, control researchers have begun to apply alternative methodologies to flight control design. One alternative is variously called non-linear dynamic inversion or feedback linearization.

The purpose of dynamic inversion is to develop a feedback control law that linearizes the plant response to commands. Simple controllers can then be designed.

With dynamic inversion, a non-linear control law is designed which globally reduces the dynamics of selected controlled variables to integrators. A closed loop system is then designed to make the controlled variables exhibit specified command responses while satisfying the usual disturbances response and robustness requirements for the overall system and the various physical limitations of the aircraft's control effectors.

### 4.1.2 System model

In general the nonlinear aircraft dynamics can take the form:

$$\frac{dx}{dt} = F(x, u)$$
$$y = H(x)$$

Where the symbols, F(.,.) and H(.) denote non-linear functions, known with reasonable accuracy, that are a mix of analytic expressions and tabular data, x is an n-dimensional state vector, u, is an $n_u$-dimensional input, y is an $n_y$-dimensional vector of output variables.

For rigid body aircraft dynamics, it is possible to rewrite the state equations in the following form (by means of appropriate changes of variables):

$$\begin{gathered} \frac{dy}{dt} = y_1 \\ \frac{dy_1}{dt} = y_2 \\ \vdots \\ \frac{dy_m}{dt} = f_y(x, u) \\ \frac{dz}{dt} = f_z(x, u) \end{gathered}$$

where $(y, y_1, y_2, ..., y_m, z)^T$ is a transformed version of the original state vector.

To obtain this alternative form formally, we can time-differentiate y(x) and define its derivative expression to be a new function, $y_1(x)$. Next, we time-differentiate $y_1(x)$ and define its derivative expression to be $y_2(x)$, then we time-differentiate $y_2(x)$ and define $y_3(x)$, etc. We stop when the control variables, u, appear explicitly in the next time-derivative. Then, as a final step, we must complete the state transformation by selecting a function, z(x), which fills out $(y, y_1, y_2, ..., y_m, z)^T$ and makes it invertible for all x.

As a result, we get a single three-element equation

$$\frac{dy}{dt} = \left[ \frac{\partial H}{\partial x^T} \right] F(x, u)$$

Furthermore, the right-hand side of this equation turns out, in most cases, to be linear in u, thus yielding the form,

$$\frac{dy}{dt} = f(x) + g(x)\, u$$

with g(x) invertible for all values of x. f(x) represents the non-linear output dynamics and g(x) represents the non-linear control distribution.

### 4.1.3 Controller structure

The final control laws obtained from dynamic inversion consist of a non-linear block which performs the feedback linearization and a controller block which implements the compensator.

The inverse dynamics control law can be written as:

$$u = g(x)^{-1} [v - f(x)]$$

The parameter v represents the desired linear dynamics of the closed-loop system. With the inverse dynamics control law implemented, the closed-loop system now has the form

$$\frac{dy}{dt} = v$$

Figure 4.1.3./1. shows the above ideas.

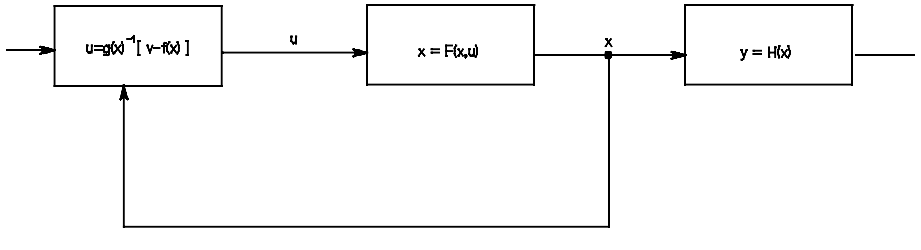

Figure 4.1.3.1./1 Close-Loop Configuration using Non-Linear Dynamic Inversion Technique.

### 4.1.4 Design specifications

The basic design specification in this method are

- to have good responses for pilot commands. This usually takes the form of equivalent linear system models for the primary commanded variables, with unmodelled parameters constrained to fall into specified ranges.

- to attenuate undesirable external disturbances.

One important aspect of the dynamic inversion concept is the presence of hidden zero dynamics. These dynamics are implicitly defined by the selected controlled variables, and they must be examined separately to make certain that they are stable and well-behaved.

Furthermore, good nominal response and well-behaved zero dynamics are not enough for the controller design. These qualities must also be robust with respect to various modelling errors inherent in aircraft systems.

### 4.1.5 Robustness Properties

The major dynamic inversion robustness issues are exhibited by replacing the nominal aircraft model with a perturbed model, i.e.

$$\frac{dy}{dt} = (f + \delta f) + (g + \delta g)\, u$$

And also replacing the ideal control effector position with a perturbed value obtained by passing the ideal position through actuator dynamics, flexible structural elements, and other high-frequency uncertainties, i.e.

$$u = (I + \Delta)\, g(x)^{-1}\, (v - f(x))$$

Enns et al. (1994) discuss robustness properties.

### 4.1.6 Control Synthesis Methodology

Some key aspects of this implementation include on-board models and full state feedback. Nevertheless, aircraft surface allocation and limiting logic (e.g. daisy-chain and anti-windup logics) can be easily implemented.

The design method is summarised in:

Step 0: Express aerodynamic tabular data as non-linear functions.

Step 1: Reformulate, if necessary, the original system to obtain an approximate nonlinear model from which a state-dependent nonlinear inverse can be easily implemented.

Step 2: Calculate the dynamic inverse feedback controller; then use the feedback control to obtain y'= v.

Step 3: Design a linear feedback control law based on y'= v to achieve the desired objectives.

(See Lin 1994)

## 4.2 Example of Application

The main application areas are supermanoeuverability and high-angle-of-attack.

In regimes where angle of attack is high, non linearities become a predominant feature of aircraft dynamics. Purely linear controllers are not able to effectively control supermanoeuverable aircraft for more than very limited flight envelopes. This limitation has motivated the exploration of non-linear techniques such as dynamic inversion (or feedback linearization).

Bugajski and Enns (1992) have used non-linear dynamic inversion to control the high-angle-of-attack research vehicle (HARV) across a wide, high-angle-of-attack flight envelope. The control laws, as well as the aircraft model, use a full non-linear aero database. They have also implemented an algorithm to handle appropriate surface limiting while ensuring a solution to the iterative process inherent in the dynamic inversion methodology. Via simulation they have shown that the control laws exhibit excellent tracking performance.

Snell et al. (1992) compared the performance of a dynamic inversion control system to one designed using conventional gain scheduling. Their conclusions are, the dynamic control laws are superior to gain-scheduled control laws in providing accurate control of sideslip and lateral acceleration and in reducing control deflections. For a similar level of design effort the inversion control laws provided much better dynamic response. However, questions remain with regard to robustness properties of the dynamic inversion control laws.

Huang et al. (1990) have used a dynamic inversion approach to develop high-angle-of-attack control laws for the X-29 aircraft.

In Adams et al. (1994) the method of dynamic inversion and structured singular value synthesis are combined into an approach which addresses both the nonlinearity and robustness problems of flight at extreme operating conditions. The design goals are achieved across a broad range of airspeeds, altitudes, and angle of attack. High-fidelity simulations show that the nonlinear aspects of the control laws perform well in a highly dynamic, nonlinear environment.

Enns et al. (1994) describes nonlinear dynamic inversion as an alternative design method for flight controls.

In Khan and Lu (1994) the control law is constructed based on minimization of local errors between the controlled variables and their desired values.

In Durham et al. (1994) a nonlinear model-following control design is applied to the problem of control of six degrees of freedom.

## 4.3 Computational Aspects

No available information

## 4.4 Comparative Study

Advantages:

Greater generality for re-use across different airframes, greater flexibility for handling changing models as an airframe evolves during its design cycle, and greater power to address non-standard flight regimes such as supermanoeuvrability.

Furthermore, gain scheduling is not required to ensure the flight control system stability over the entire operation envelope of the aircraft.

Drawbacks:

High order controller, possibility of control saturation, possibility of unstable zero-dynamics, large amounts of processing power.

Limitations:

This approach is limited to minimum phase systems, since when applying this technique to non minimum phase systems, the closed loop system could become unstable. More precisely, the dynamic inversion control law will induce unobservable subdynamics (the zero dynamics). Then, when the zero dynamics are unstable, the closed loop under the inversion control is unstable too.

However, for slightly non-minimum phase nonlinear systems, designing a feedback control based on a minimum phase approximation to the true system can result in a system with better properties than the one obtained from a straightforward application of the linearization theory.

## 4.5 Applicable References

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Enns et al. (1994). "Dynamic inversion: an evolving methodology for flight control design". Int. J. control vol. 59, no1. pp 71-91.

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# 5 Lyapunov Methods

Due to the strong mathematical character of Lyapunov methods, two different descriptions are included. The first one contains a basic introduction to the Lyapunov methods for the stability analysis of dynamic systems. The second one deals with a more detailed description in which the main characteristics of the method, as well as examples of application to robust control problems, are included.

## 5.1 Basic method description

This Lyapunov method description contains a basic introduction (first level) to the methodology.

### 5.1.1 Introduction

Stability is the most important property for control systems. The Lyapunov method is the most general method for stability analysis of non-linear and/or time-varying systems.

Following this method the stability of a system is determined without solving the dynamic problem. This is a very important advantage, since for non-linear and/or time-variant systems the solution of the dynamic problem can be very difficult. Also, the method is applicable whatever the order of the system.

Although a large amount of experience can be required to apply the method, it can be used to analyze the characteristics of a system when other methods fail.

The basic idea of the method is to look for a function (called the Lyapunov function) that is related to the amount of excess energy that the current dynamic state has with respect to the energy of the analyzed equilibrium state. If this function is found to be a bounded, monotonic decreasing function then stability can be guaranteed.

Although these methods are mainly devoted to the study of the stability characteristics of the equilibrium solutions for dynamic systems, they have also been used to define feedback controllers that provide equilibrium solutions with the required stability characteristics.

By defining the dynamics of perturbations around a commanded evolution these methods allow the analysis of the characteristics of these dynamics and consequently lead to the definition of controllers that provide the required behaviour.