Control no lineal de vehículos marinos subactuados no-holonómicos

Dictino Chaos, Joaquín Aranda, David Moreno, Alejandro Moreno

Departamento de Informática y Automática
Escuela Técnica superior de Ingeniería Informática
Universidad Nacional de Educación a Distancia

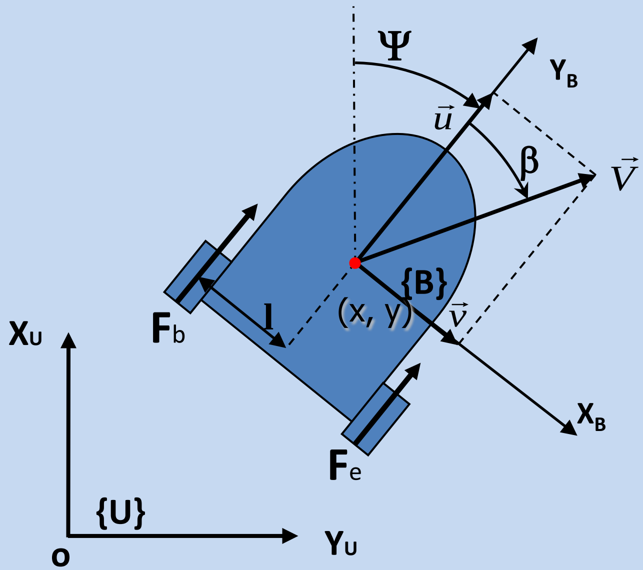
Outline

- Problem analysis
- Experimental setup
- Point Stabilization with discrete inputs
- Trajectory tracking with discrete inputs

- Platform for maritime control testing
  - A Hovercraft eliminates the need of water
  - Ship-like dynamics
  - Nonlinear system with nonholonomic restrictions
  - Vehicle with **drift**
- Control problems
  - Tracking
  - Point stabilization

Problem analysis

- System model
  - 3 DOF (x,y,ψ)
  - Inertial and rotating frames
  - Dynamic equation
    - Isotropy
    - Constant parameters



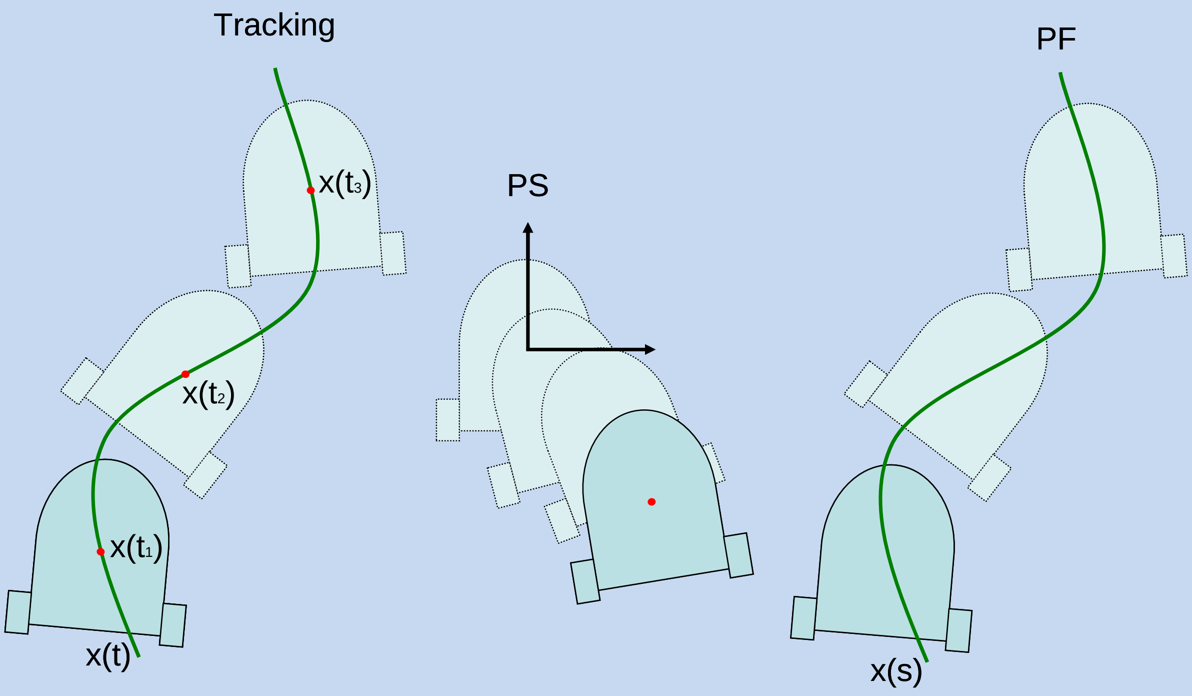
$$m\dot{u} - mrv + D_u u = F_b + F_e$$
$$m\dot{v} + mru + D_u v = 0$$
$$J\dot{r} + D_r r = l(F_b - F_e)$$

$$\dot{x} = v_x$$
$$\dot{y} = v_y$$
$$\dot{v}_x = F\cos(\psi) - d_u v_x$$
$$\dot{v}_y = F\sin(\psi) - d_u v_y$$
$$\dot{\psi} = r$$
$$\dot{r} = \tau - d_r r$$

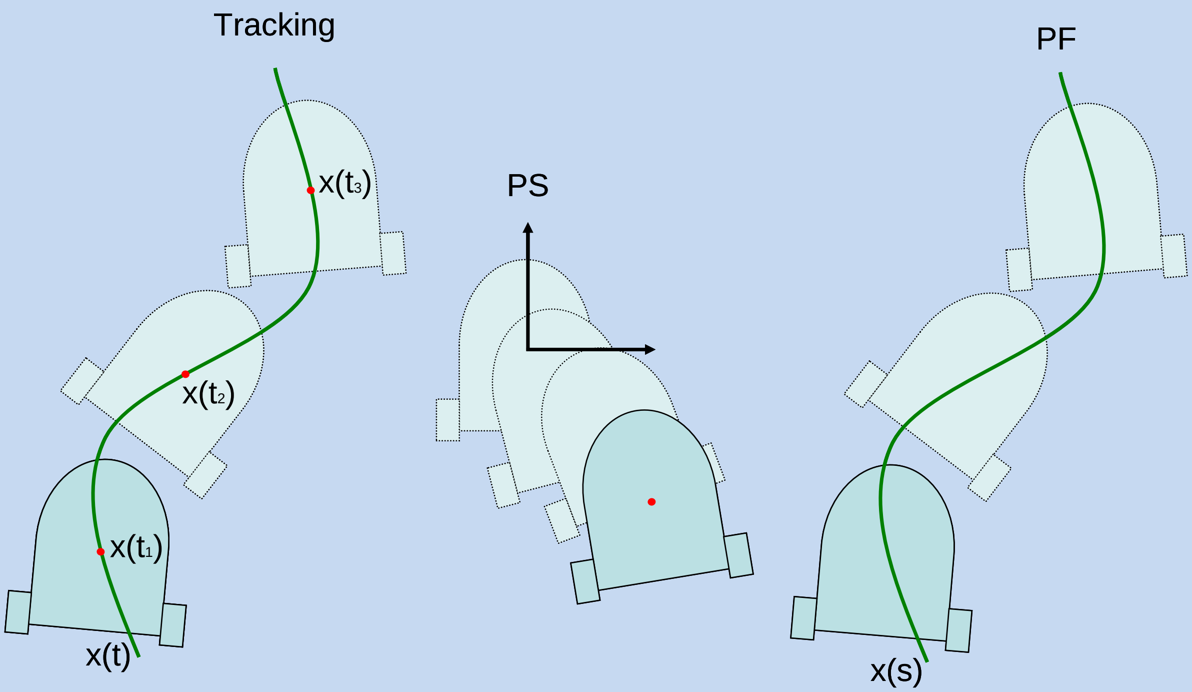
Where $F=(F_b+F_e)/m$ and $\tau=l(F_b-F_e)/J$

$d_u=D_u/m$ y $d_r=D_r/J$ are normalized drags (s^{-1})

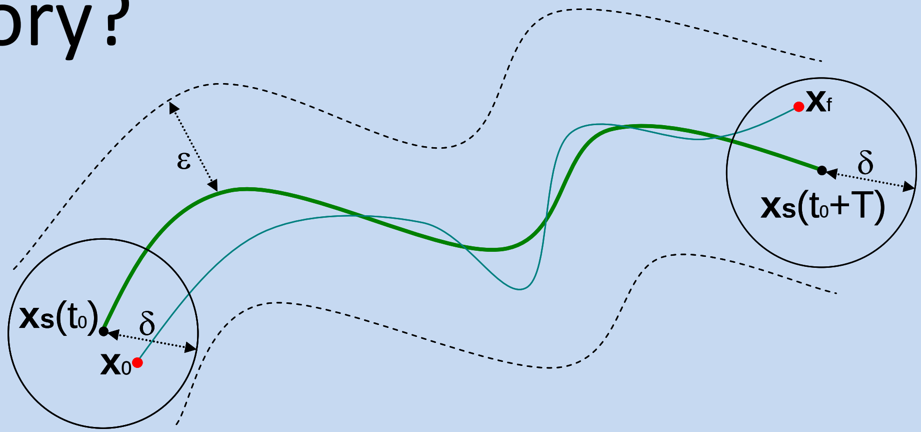
- Trajectory tracking
- Point stabilization
- Path following



- Trajectory tracking
- Point stabilization
- Path following



- Can we move the state between points close to the trajectory?



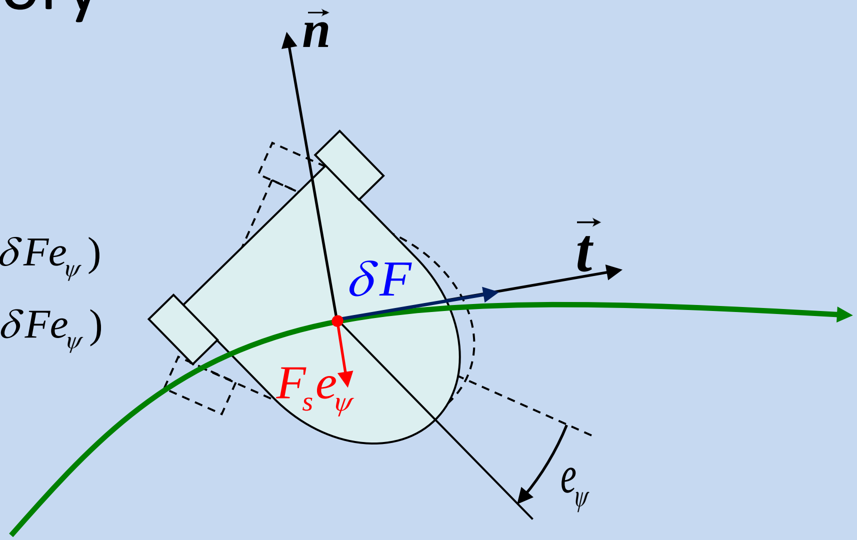
- System controllable if for some time *t*

$$F_s(t)^2+\left(d_u{}^2 r_s(t)^2+4r_s(t)^4+2r_s(t)\ddot{r}_s(t)-3\dot{r}(t)_s{}^2\right)^2\neq 0$$

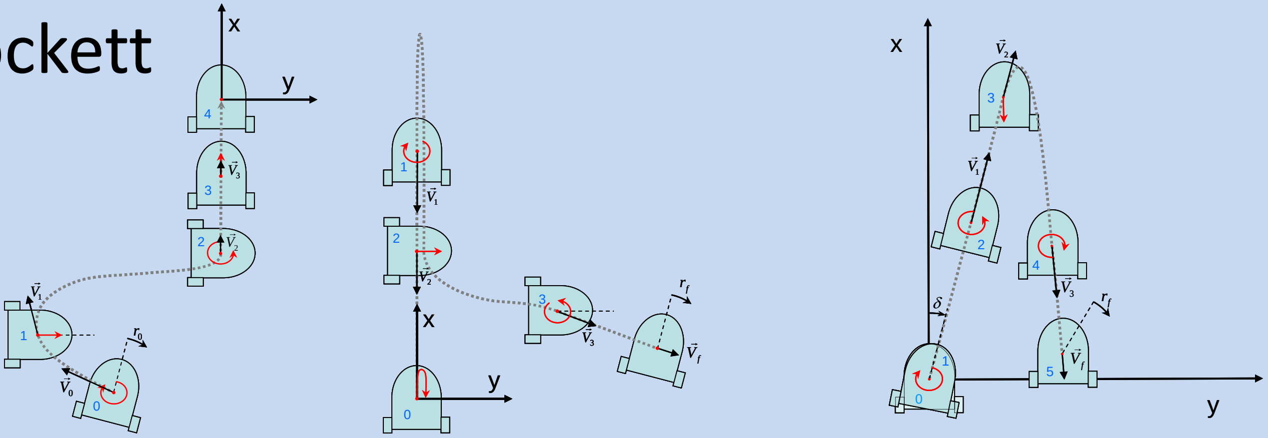
  - Controllable on any non-free trajectory
  - Controllable on any non-stopping trajectory

- Intuitive explanation
  - Modifying force gives a control action tangent to trajectory
  - The error in orientation gives some extra control action orthogonal to trajectory

$$\begin{aligned}
\dot{e}_x &= e_{vx} \\
\dot{e}_y &= e_{vy} \\
\dot{e}_{vx} &= -d_u e_{vx} + \delta F \cos(\psi_s) - F_s \sin(\psi_s) e_\psi + o(e_\psi{}^2, \delta F^2, \delta F e_\psi) \\
\dot{e}_{vy} &= -d_u e_{vy} + \delta F \sin(\psi_s) + F_s \cos(\psi_s) e_\psi + o(e_\psi{}^2, \delta F^2, \delta F e_\psi) \\
\dot{e}_\psi &= e_r \\
\dot{e}_r &= -d_r e_r + \delta\tau
\end{aligned}$$



- What happens if *F*=0?
  - Test LARC condition (lie algebra rank condition)
    - Drift cannot be removed (system cannot stop with $v \neq 0$)
    - LARC only implies that states are connected by trajectories
  - There exist control sequences (controllable)
  - There exist local sequences to approach to a equilibrium point (local controllable)
  - Brockett



Experimental setup

Experimental setup

Model identification

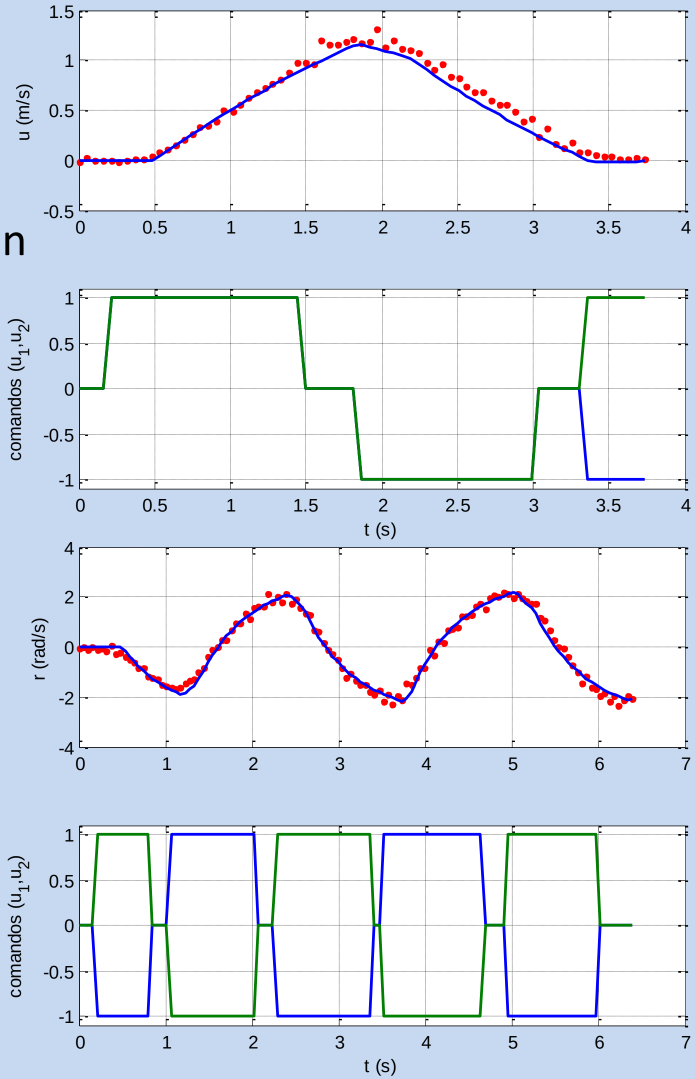
Experimental setup

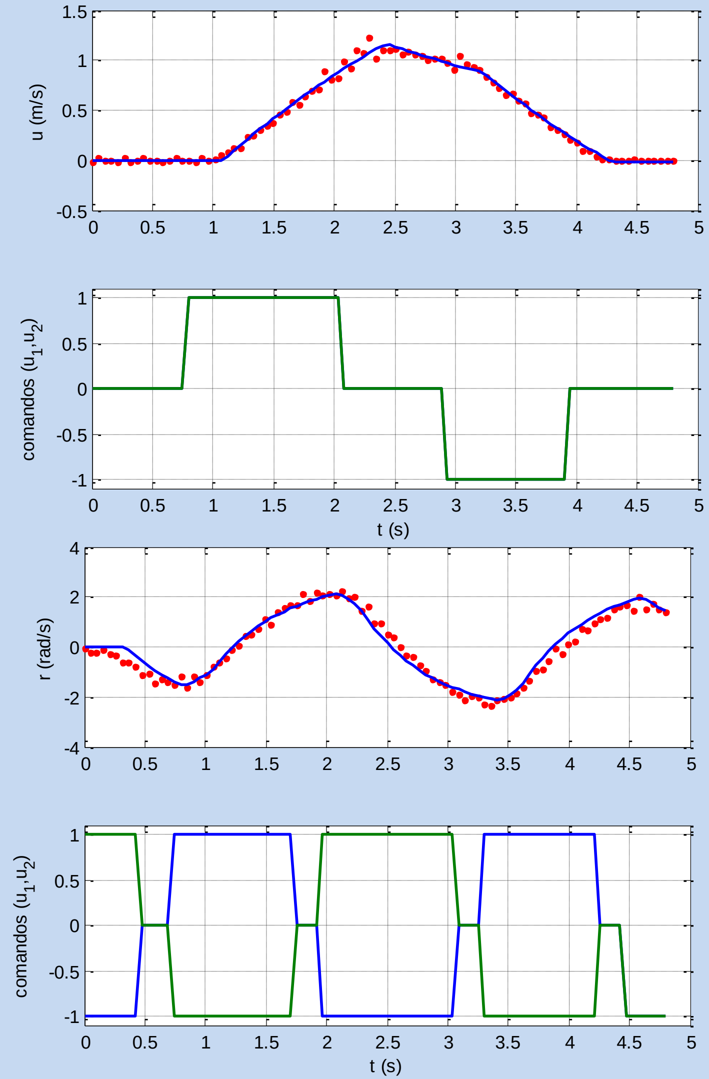
Identification

Validation

Translation

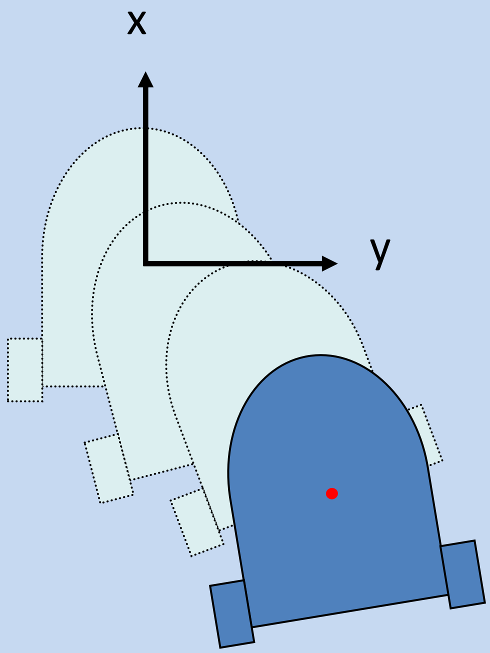
Rotation





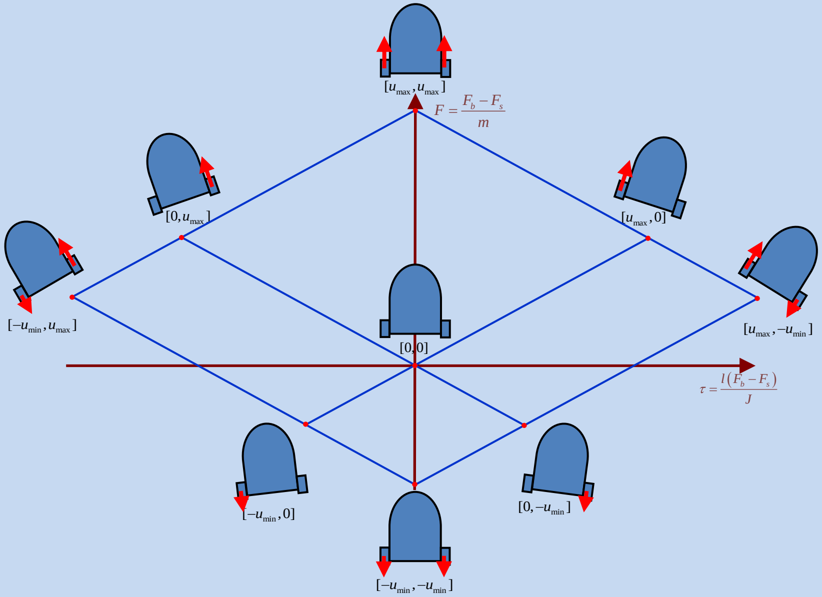
Point stabilization

- Goals
  - Stabilize the hovercraft on the origin of coordinates aligned with ***X*** axis
  - Use a discrete set of inputs
  - Robust against bound disturbances and noise



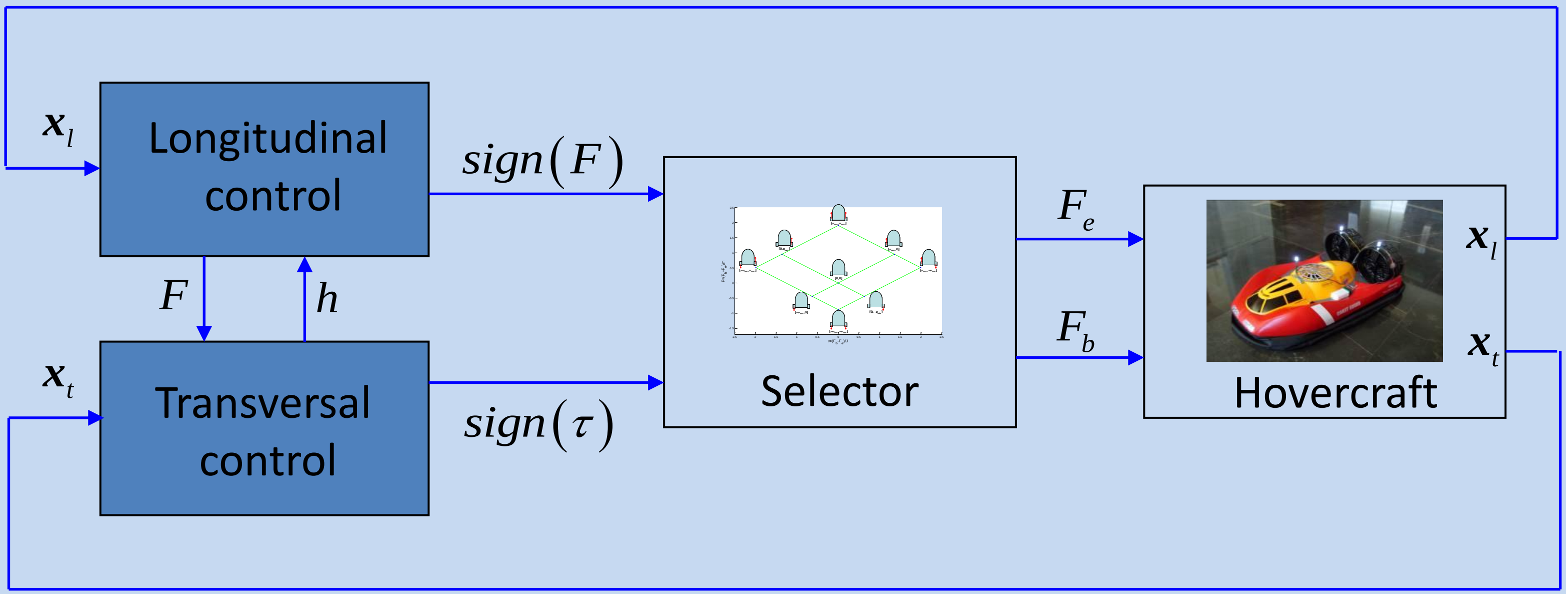
$$\sup_{t\geq 0} \|\boldsymbol{p}\| \leq p_{\max}$$
$$\sup_{t\geq 0} \|\boldsymbol{n}\| \leq n_{\max}$$

- On-off-reverse motors
- Force and torque lies in a discrete set
- Al the possible combinations or force and torque signs can be allocated



	τ_i>0	τ_i<0
F_i>0	$F_b = u_{max}$ $F_e = -u_{min}$	$F_b = -u_{min}$ $F_e = u_{max}$
F_i<0	$F_b = 0$ $F_e = -u_{min}$	$F_b = -u_{min}$ $F_e = 0$

- Discompose problem in two sub problems
  - Longitudinal x, v_x using force
  - Transversal y, ψ, v_y, r using torque
  - Interconnection using dynamic hysteresis $h(t)$



- Assume that *F* is constant
  - Define e_1 to control *y*, ψ

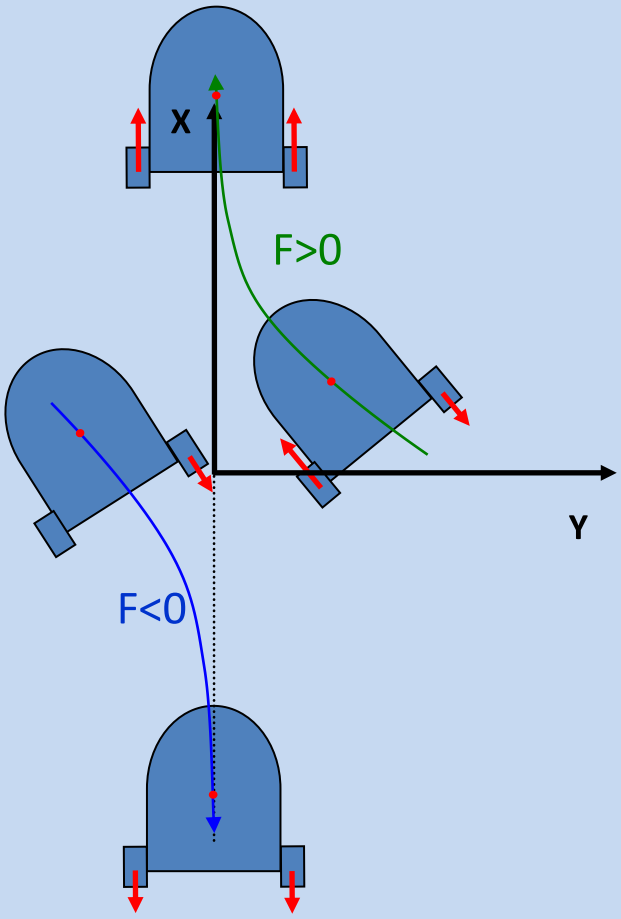
$$e_1 = \psi + k_1 sign(F)\tanh(y)$$

$$e_2 = k_2 e_1 + \dot{e}_1$$

$$\dot{e}_2 = \tau + (k_2 - d_r)r + p_r + k_1 g(k_2, y, v_y, p_{vy}, \psi)$$

- Control law

$$sign(\tau) = \begin{cases} -sign(e_2) & si \ \ |e_2| \geq \varepsilon_1 \\ sign(\tau^-) & si \ \ |e_2| < \varepsilon_1 \end{cases}$$



- *e_2 converges in finite time*

$$V_1 = \frac{e_2^2}{2} \longrightarrow \dot{V}_1 \leq -|\tau_i||e_2|\alpha = -|\tau_i|\alpha\sqrt{2V_1} \longrightarrow |e_2(t)| \leq \max\left(cn_{\max}, \varepsilon_1, |e_2(0)| - \tau_i \alpha t\right)$$

- Due to noise actually converge to a small value

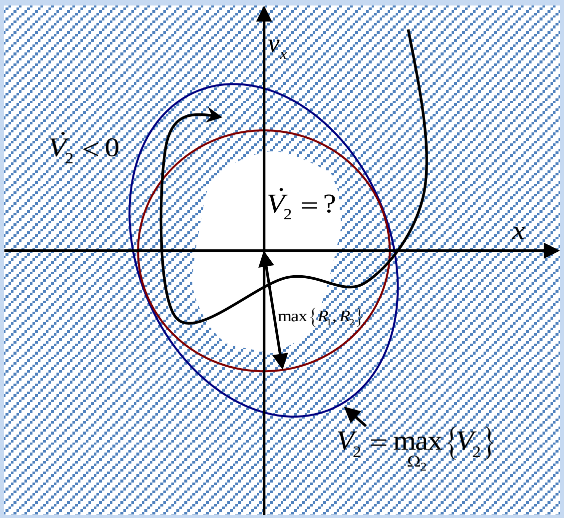
$$|e_2(t)| \le \max\left(cn_{max}, \varepsilon_1, |e_2(0)| - \tau_i \alpha t\right)$$

- This implies that e_1 also converge

$$\dot{e}_1 = -k_2 e_1 + e_2$$

- what happens with y?

$$\dot{y} = v_y$$
$$\dot{v}_y = -d_u v_y - |F| \sin(k_1 \tanh(y) - sign(F) e_1) + p_{vy}$$



Radically unbounded definite positive Lyapunov function

$$V_2 = \frac{d_u}{4} y^2 + 2|F| \int_0^y \sin\left(k_1 \tanh(s)\right) ds + \left(v_y + \frac{d_u y}{2}\right)^2$$

$$R = \sqrt{y^2 + v_y^2} \ge R(\mathbf{n}, \mathbf{p}) = \gamma\left(\|\mathbf{n}\| + \|\mathbf{p}\| + \varepsilon_1\right)$$

$$\dot{V}_2 \le -d_u v_y^2 - d_u |F| y \sin\left(k_1 \tanh(y)\right) + |2v_y + d_u y|\left(|F||e_1| + |p_{vy}|\right) \le 0$$

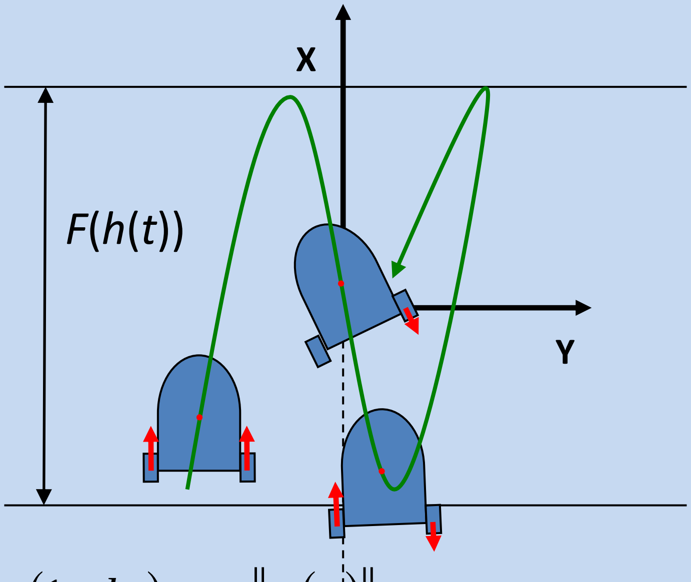
- Transversal dynamics is globally finally bounded

- Tries to maintain *x* in a *h* band of the origin

$$e_3 = x + k_3 v_x$$

$$\dot{e}_3 = (1 - k_3 d_u) v_x + k_3 F \cos(\psi) + k_3 p_{vx}$$

$$sign(F) = \begin{cases} -sign(e_3) & si \quad |e_3| \geq h(t) \\ sign(F^-) & si \quad |e_3| < h(t) \end{cases}$$



- Stability ($|e_3| > h, n$)

$$V_3 = \frac{e_3^2}{2} \longrightarrow e_3(t) \leq \beta\left(|e_3(t_0)|, t - t_0\right) + \sup_{t_0 \leq \tau \leq t} |h(\tau)| + (1 + k_3) \sup_{t_0 \leq \tau \leq t} \|\mathbf{n}(\tau)\|$$

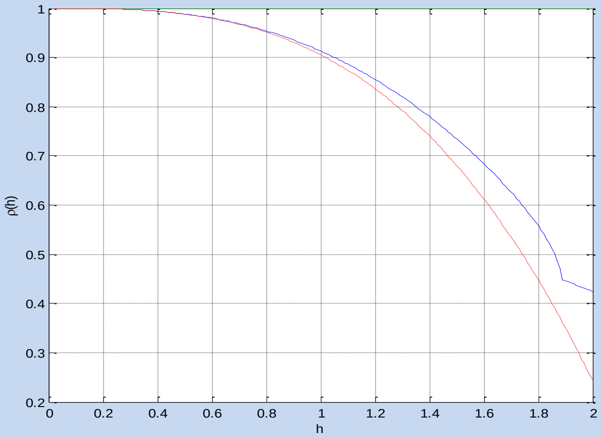
$$\dot{x} = v_x = \frac{-x + e_3(t)}{k_3} \longrightarrow |x(t)| \leq |x(t_0)| e^{-\frac{\varepsilon}{k_3}(t - t_0)} + (1 + \varepsilon)|e_{3m}|$$

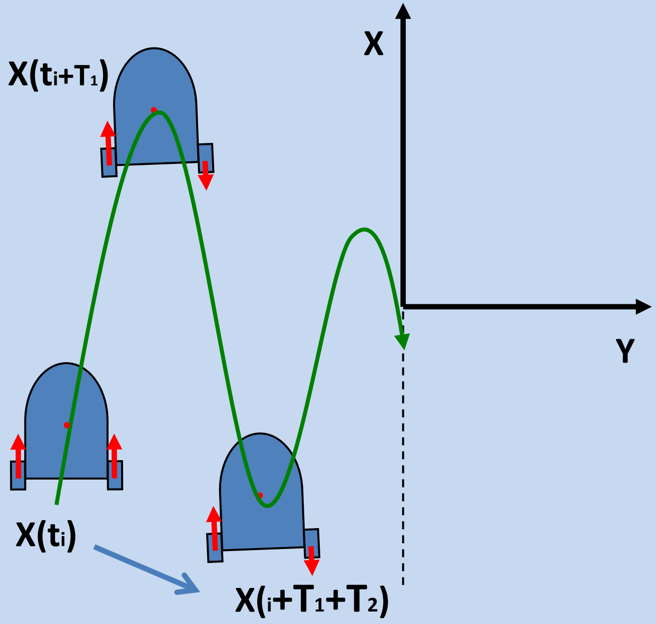
- $\boldsymbol{x}_t$ is ISS respect to noise an *h*

- The main problem is to make $h \rightarrow 0$ when $\boldsymbol{x}_t \rightarrow 0$
- Study the equivalent system
  - Discrete system in on the odd transitions
  - Linearization considering e_3 as a disturbance
- Overall system is ISS respect to $n_{\max}$ y $p_{\max}$

$$\boldsymbol{x}_{n+1} = \boldsymbol{A}_{eq}(h_n)\boldsymbol{x}_n + \boldsymbol{d}_n$$

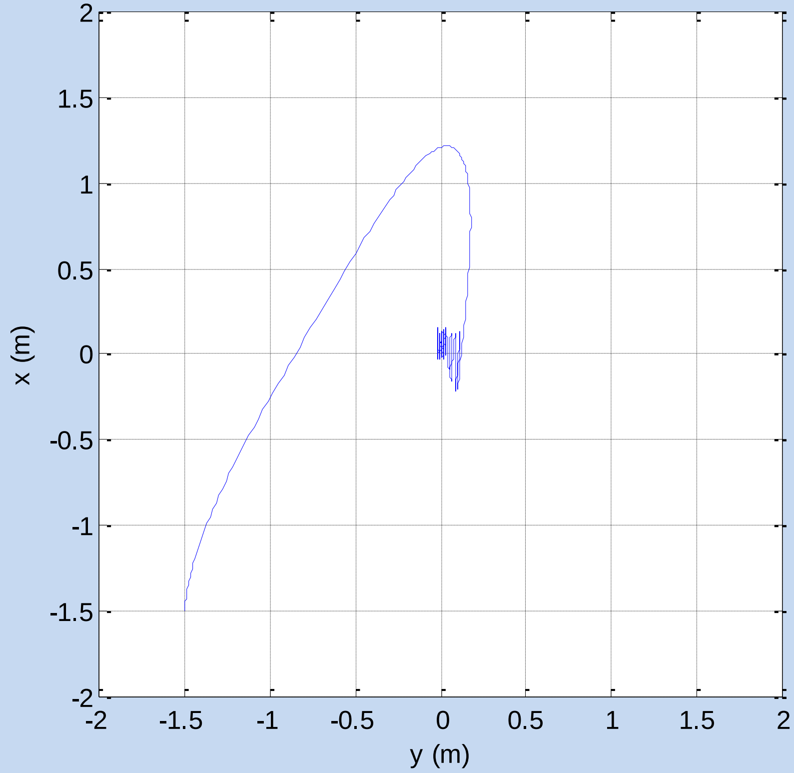
$$\|\boldsymbol{d}_n\| \le c_1\|\boldsymbol{x}_n\|^2 + c_2|\Delta h_n|\|\boldsymbol{x}_r\| + \gamma_1(\max(n_{\max}, p_{\max}))$$



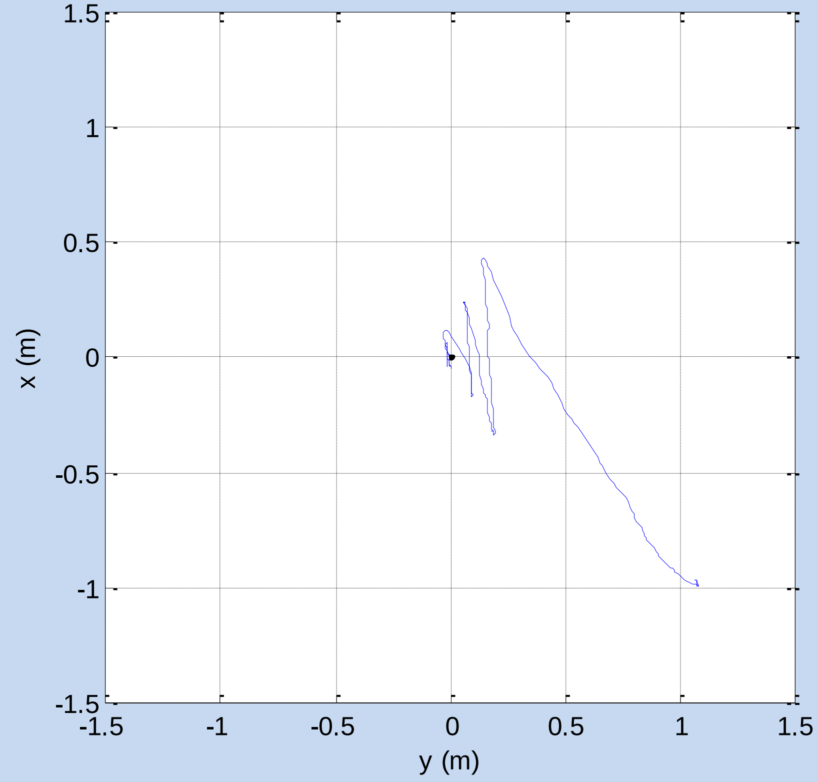


- $k_1 = k_2 = k_3 = 1,\ \varepsilon = 0.001 \quad h = \sqrt{\|\mathbf{x}_r\|}$

Simulation



Experiment

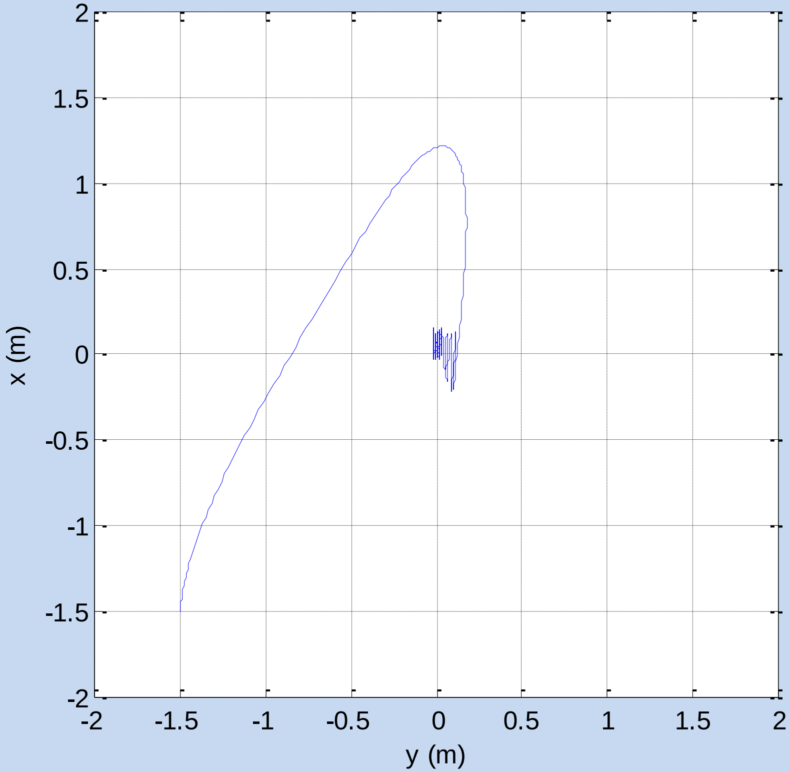


Experimental results — Point stabilization

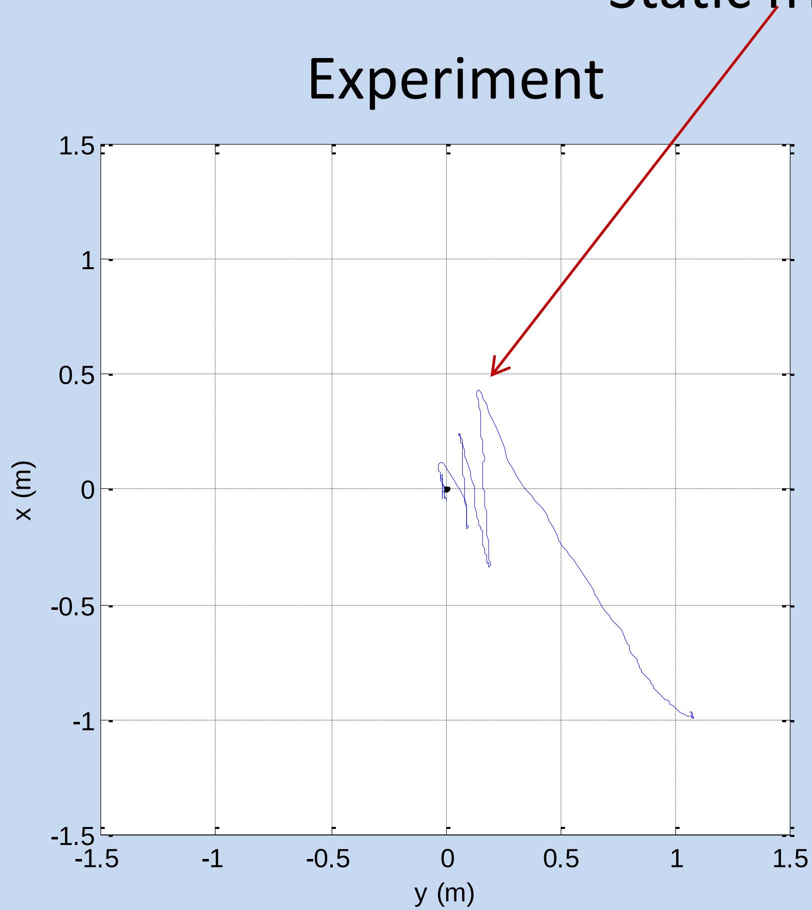
- $k_1=k_2=k_3=1,\ \varepsilon=0.001 \qquad h=\sqrt{\lVert \mathbf{x}_r \rVert}$

Static friction

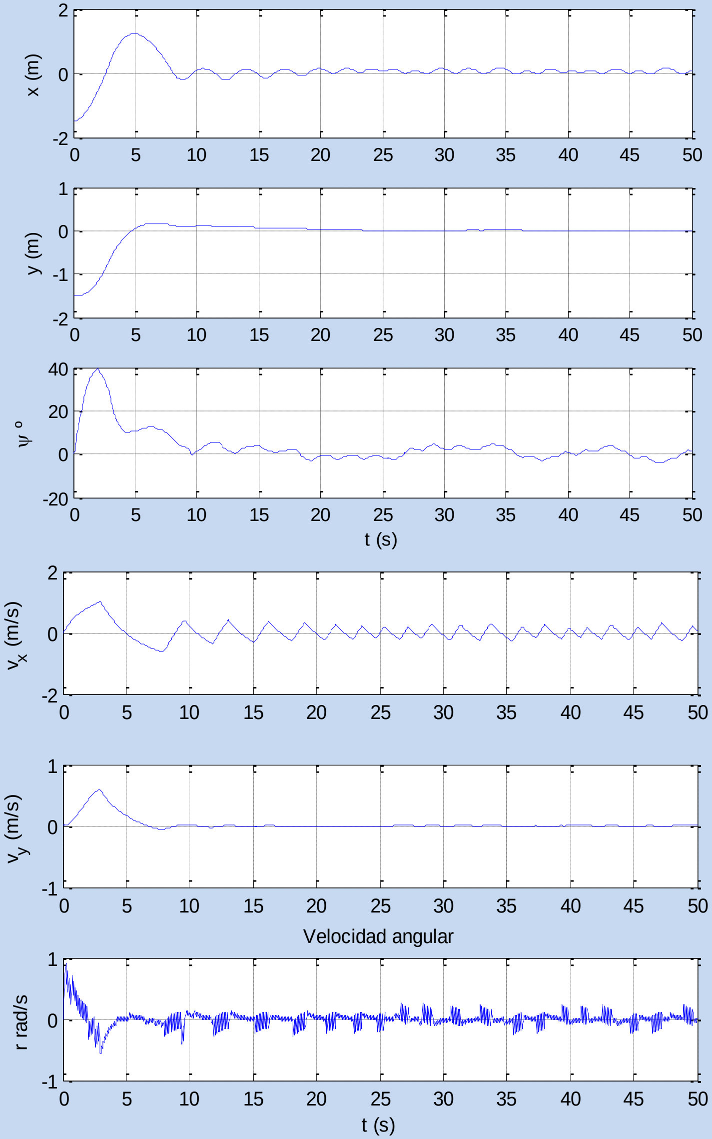
Simulation



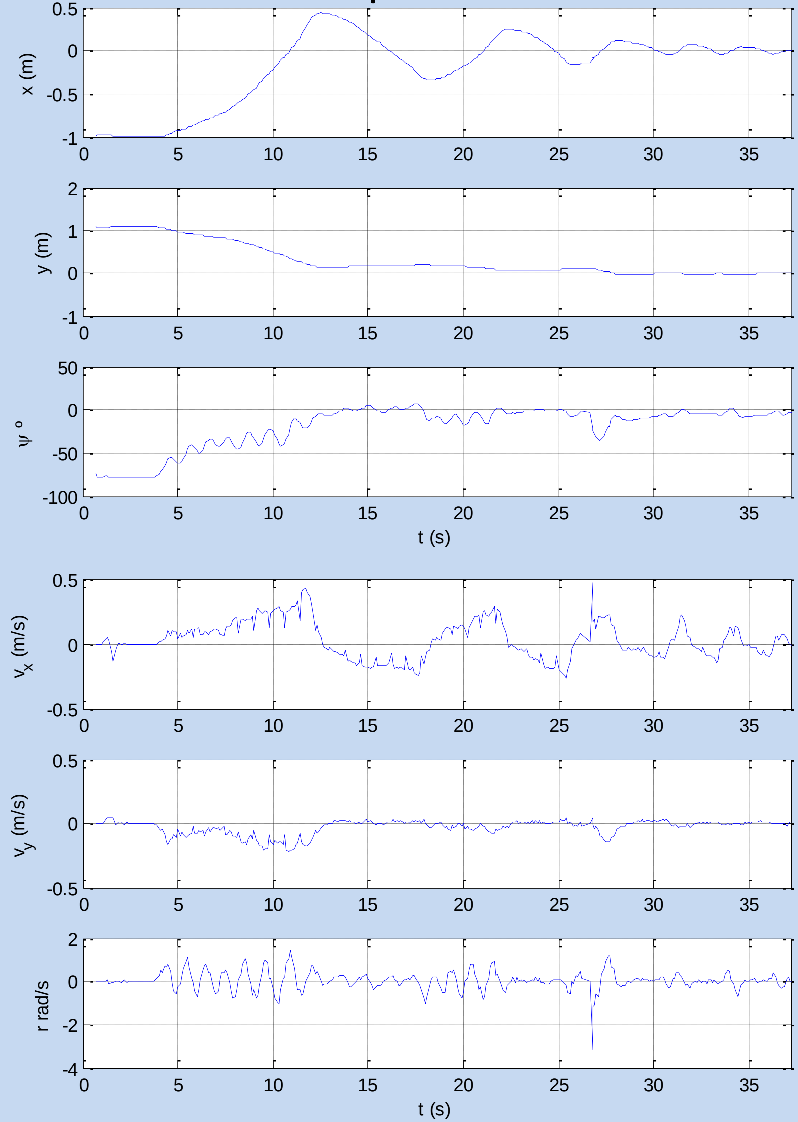
Experiment



Simulation

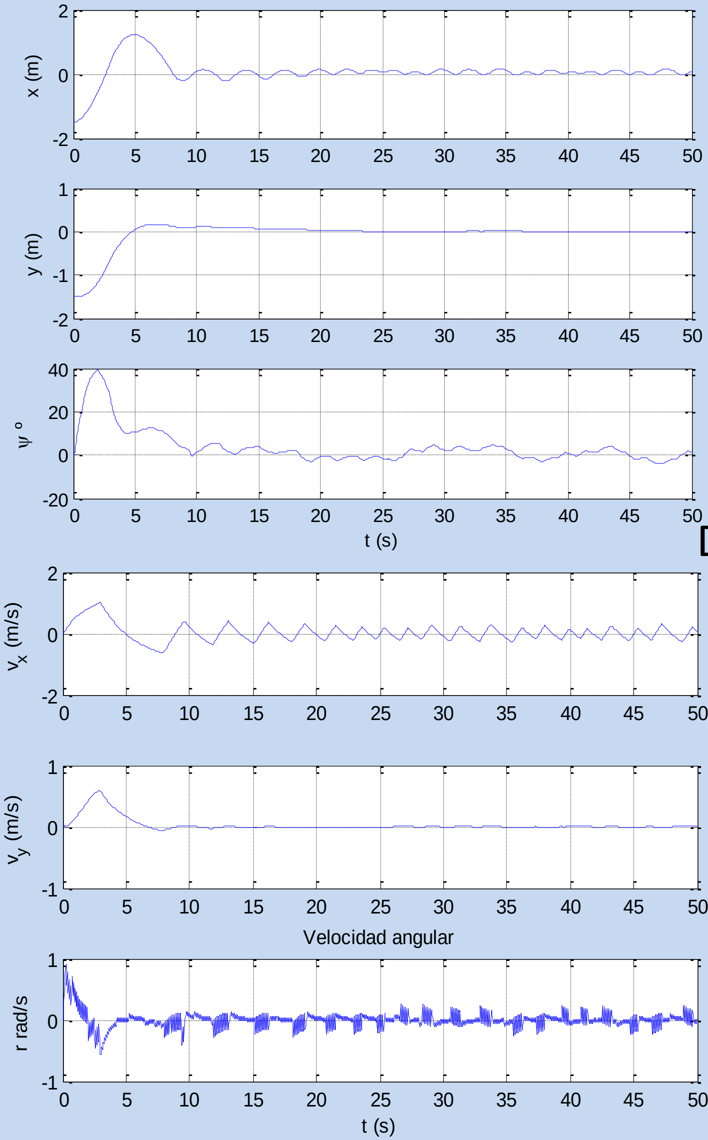


Experiments

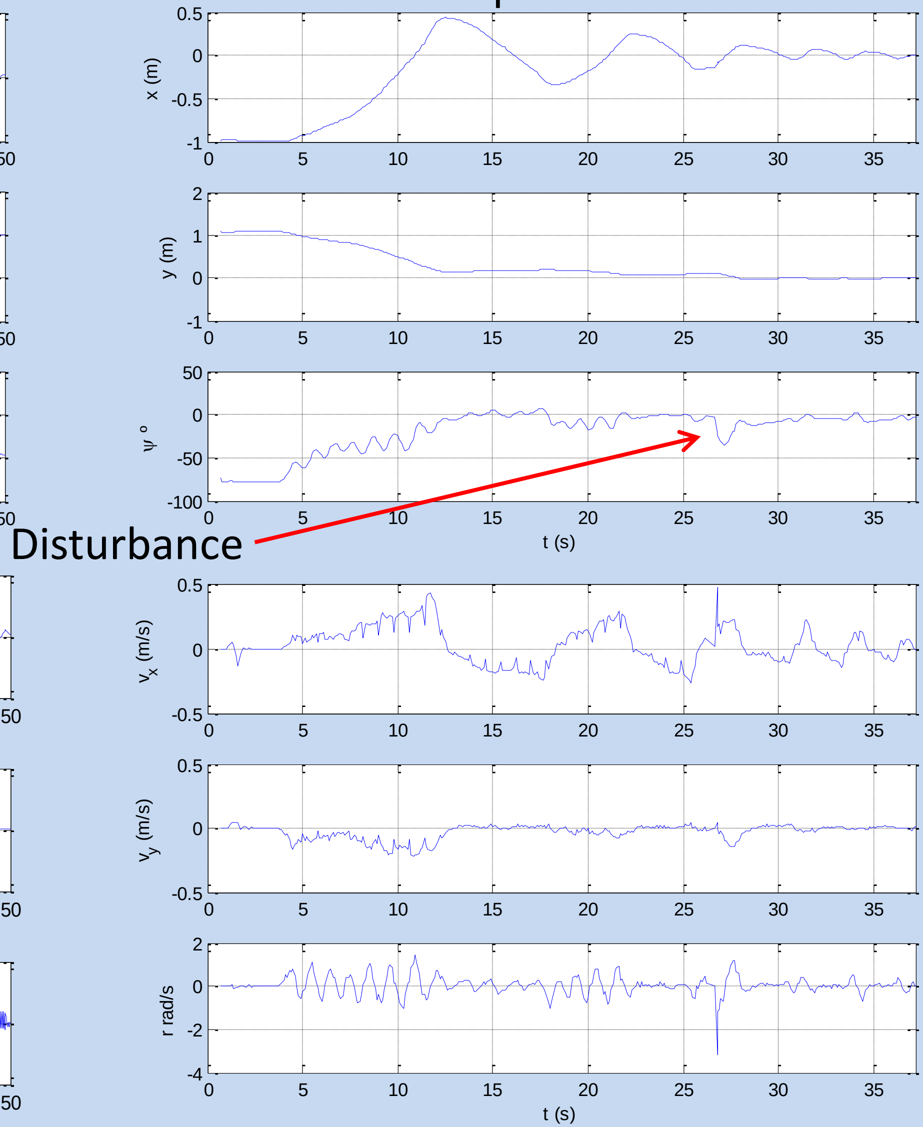


Point stabilization

Simulation



Experiments



Video

Discrete tracking

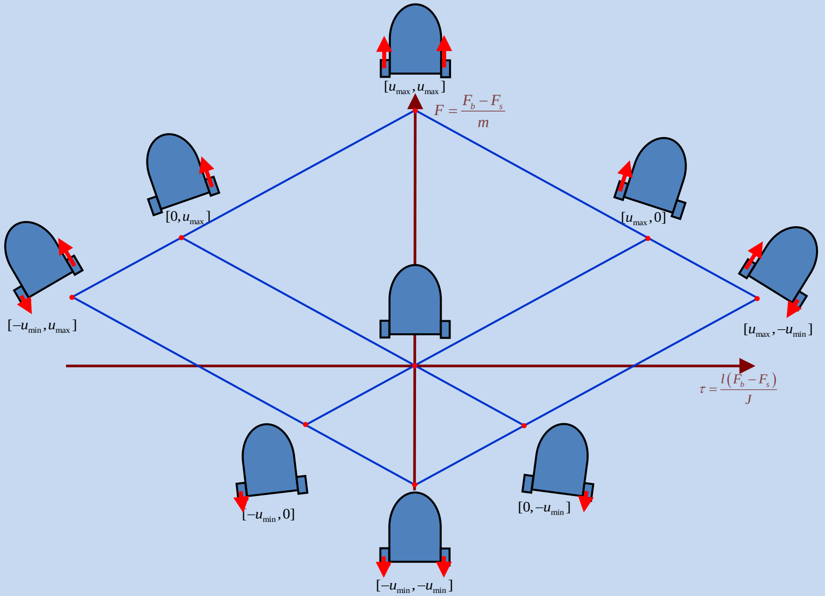
- Goals
  - Track a reference spatial trajectory $x(t),y(t)$
  - Use only a discrete set of control inputs

    $F_{b,e} \in \{-u_{min}, 0, u_{max}\}$
  - Robust against bounded noise and disturbances

$$\sup_{t\geq 0} \|\boldsymbol{p}\| \leq p_{\max}$$

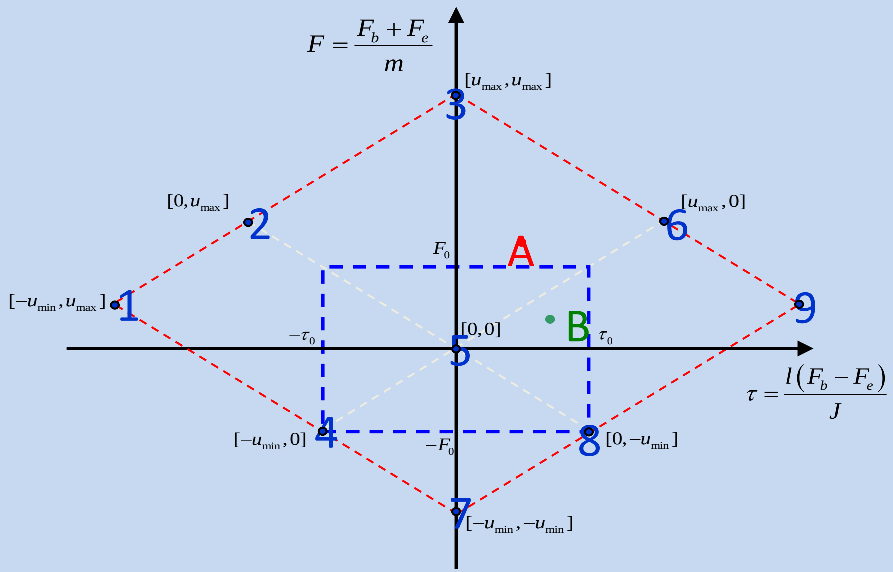
$$\sup_{t\geq 0} \left\|\frac{d\boldsymbol{p}}{dt}\right\| \leq p_{\max}$$

$$\sup_{t\geq 0} \|\boldsymbol{n}\| \leq n_{\max}$$



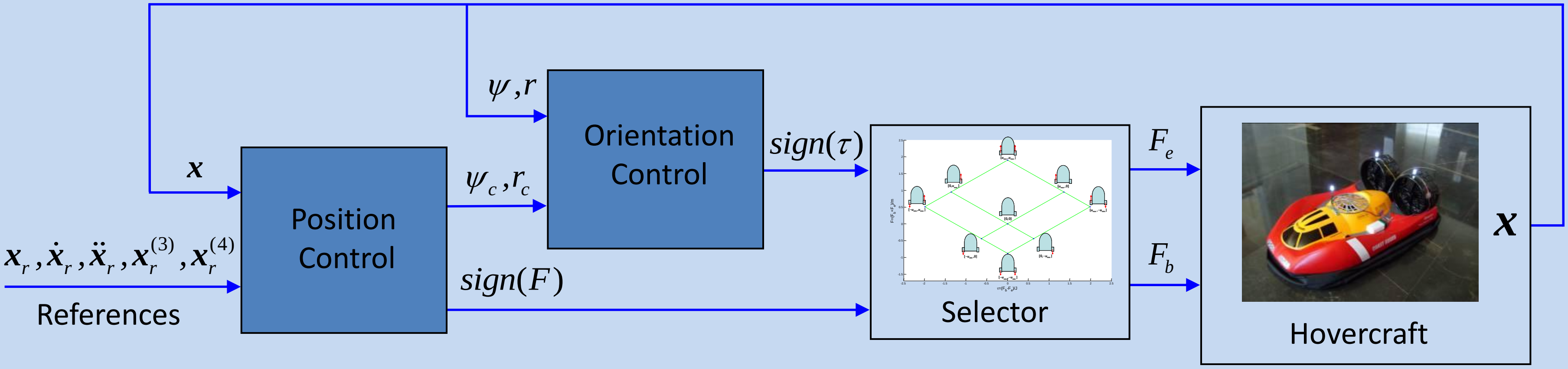
- Not all the continuous feasible trajectories can be tracked.
  - On any point of the trajectory force and torque of any sign must be allocated in order to dominate over F_r and τ_r

$$|F_r(t)| < F_i, \ |\tau_r(t)| < \tau_i$$
$$|F_r(t)| < F_j, \ |\tau_r(t)| < -\tau_j$$
$$|F_r(t)| < -F_k, \ |\tau_r(t)| < \tau_k$$
$$|F_r(t)| < -F_l, \ |\tau_r(t)| < -\tau_l$$



Dominable region is smaller that feascible one

- Cascade control
  - Outer loop determines the sign of *F* and the orientation references
  - Inner loop determines the sign of τ
  - Force selector to produce force an torque (like PS)



- Start with the tracking error dynamics

$$\dot{e}_x = e_{vx}$$
$$\dot{e}_y = e_{vy}$$
$$\dot{e}_{vx} = F\cos(\psi) - d_u e_{vx} - F_{xr} + p_{vx}$$
$$\dot{e}_{vy} = F\sin(\psi) - d_u e_{vy} - F_{yr} + p_{vy}$$

- Start with the tracking error dynamics

$$\dot{e}_x = e_{vx}$$
$$\dot{e}_y = e_{vy}$$
$$\dot{e}_{vx} = F\cos(\psi) - d_u e_{vx} - F_{xr} + p_{vx}$$
$$\dot{e}_{vy} = F\sin(\psi) - d_u e_{vy} - F_{yr} + p_{vy}$$

$$s_x = e_x + k_1 e_{vx} \quad \dot{s}_x = e_{vx} + k_1 \dot{e}_{vx} = k_1 F\cos(\psi) + \left(1 - d_u k_1\right) e_{vx} - k_1 F_{xr} + k_1 p_{vx}$$
$$s_y = e_y + k_1 e_{vy} \quad \dot{s}_y = e_{vy} + k_1 \dot{e}_{vy} = k_1 F\sin(\psi) + \left(1 - d_u k_1\right) e_{vy} - k_1 F_{yr} + k_1 p_{vy}$$

- Start with the tracking error dynamics

$$\dot{e}_x = e_{vx}$$
$$\dot{e}_y = e_{vy}$$
$$\dot{e}_{vx} = F\cos(\psi) - d_u e_{vx} - F_{xr} + p_{vx}$$
$$\dot{e}_{vy} = F\sin(\psi) - d_u e_{vy} - F_{yr} + p_{vy}$$

$$s_x = e_x + k_1 e_{vx} \quad \dot{s}_x = e_{vx} + k_1 \dot{e}_{vx} = k_1 F\cos(\psi) + (1 - d_u k_1) e_{vx} - k_1 F_{xr} + k_1 p_{vx}$$
$$s_y = e_y + k_1 e_{vy} \quad \dot{s}_y = e_{vy} + k_1 \dot{e}_{vy} = k_1 F\sin(\psi) + (1 - d_u k_1) e_{vy} - k_1 F_{yr} + k_1 p_{vy}$$

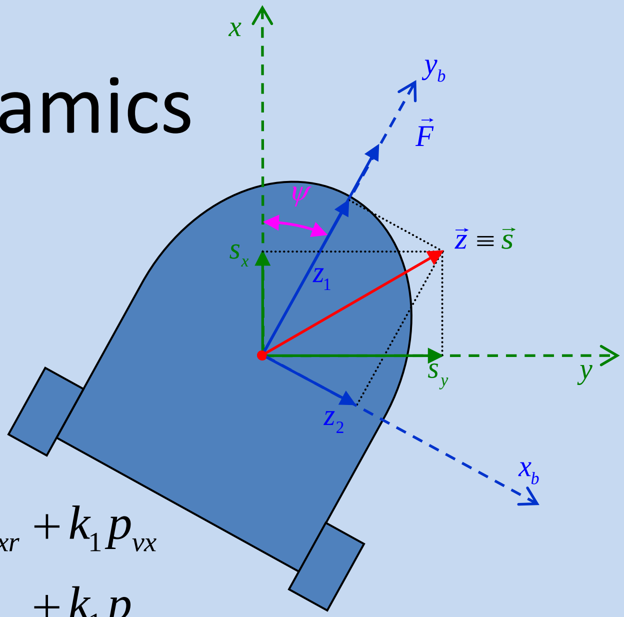
- Start with the tracking error dynamics

$$\dot{e}_x = e_{vx}$$
$$\dot{e}_y = e_{vy}$$
$$\dot{e}_{vx} = F\cos(\psi) - d_u e_{vx} - F_{xr} + p_{vx}$$
$$\dot{e}_{vy} = F\sin(\psi) - d_u e_{vy} - F_{yr} + p_{vy}$$

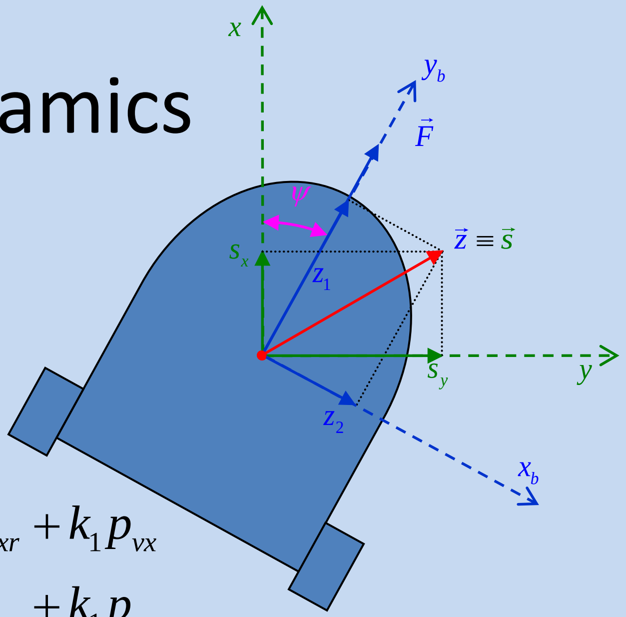
$$s_x = e_x + k_1 e_{vx} \quad \dot{s}_x = e_{vx} + k_1\dot{e}_{vx} = k_1 F\cos(\psi) + (1 - d_u k_1)e_{vx} - k_1 F_{xr} + k_1 p_{vx}$$
$$s_y = e_y + k_1 e_{vy} \quad \dot{s}_y = e_{vy} + k_1\dot{e}_{vy} = k_1 F\sin(\psi) + (1 - d_u k_1)e_{vy} - k_1 F_{yr} + k_1 p_{vy}$$

$$z_1 = s_x\cos(\psi) + s_y\sin(\psi)$$
$$z_2 = -s_x\sin(\psi) + s_y\cos(\psi)$$

$$\dot{z}_1 = k_1 F - k_1 F_r\cos(\psi - \psi_r) + z_2 r + (1 - d_u k_1)\left(e_{vx}\cos(\psi) + e_{vy}\sin(\psi)\right) + k_1 p_1$$
$$\dot{z}_2 = k_1 F_r\sin(\psi - \psi_r) - z_1 r + (1 - d_u k_1)\left(-e_{vx}\sin(\psi) + e_{vy}\cos(\psi)\right) + k_1 p_2$$



- Start with the tracking error dynamics



$$\dot{e}_x = e_{vx}$$
$$\dot{e}_y = e_{vy}$$
$$\dot{e}_{vx} = F\cos(\psi) - d_u e_{vx} - F_{xr} + p_{vx}$$
$$\dot{e}_{vy} = F\sin(\psi) - d_u e_{vy} - F_{yr} + p_{vy}$$

$$s_x = e_x + k_1 e_{vx} \quad \dot{s}_x = e_{vx} + k_1\dot{e}_{vx} = k_1 F\cos(\psi) + (1-d_u k_1)e_{vx} - k_1 F_{xr} + k_1 p_{vx}$$
$$s_y = e_y + k_1 e_{vy} \quad \dot{s}_y = e_{vy} + k_1\dot{e}_{vy} = k_1 F\sin(\psi) + (1-d_u k_1)e_{vy} - k_1 F_{yr} + k_1 p_{vy}$$

$$sign(F) = \begin{cases} -sign(z_1) & si \quad |z_1| \geq \varepsilon \\ sign(F^-) & si \quad |z_1| < \varepsilon \end{cases}$$

$$\dot{z}_1 = k_1 F - k_1 F_r \cos(\psi - \psi_r) + z_2 r + (1-d_u k_1)\left(e_{vx}\cos(\psi) + e_{vy}\sin(\psi)\right) + k_1 p_1$$
$$\dot{z}_2 = k_1 F_r \sin(\psi - \psi_r) - z_1 r + (1-d_u k_1)\left(-e_{vx}\sin(\psi) + e_{vy}\cos(\psi)\right) + k_1 p_2$$

$$\psi_c = \psi_r - k_2 \tanh(z_2)\mathrm{sign}(F_r)$$

- Definition of orientation errors

$$e_\psi = \psi - \psi_c \qquad \dot{e}_\psi = \dot{\psi} - \dot{\psi}_c = r - \dot{\psi}_c = e_r$$
$$e_r = r - \dot{\psi}_c \qquad \dot{e}_r = \tau - \tau_c - d_r e_r + p_r$$

  - New sliding variable

$$s_\psi = e_\psi + k_3 e_r$$

  - This variable is controlled using *τ*

$$\dot{s}_\psi = k_3\left(\tau - \tau_c + p_r\right) + \left(1 - k_3 d_r\right) e_r \longrightarrow sign(\tau) = \begin{cases} -sign(s_\psi) & si \quad |s_\psi| \geq \varepsilon \\ sign(\tau^-) & si \quad |s_\psi| < \varepsilon \end{cases}$$

$$sign(F) = \begin{cases} -sign(\hat{z}_1) & si \quad |\hat{z}_1| \geq \varepsilon \\ sign(F^-) & si \quad |\hat{z}_1| < \varepsilon \end{cases}$$

$$sign(\tau) = \begin{cases} -sign(\hat{s}_\psi) & si \quad |\hat{s}_\psi| \geq \varepsilon \\ sign(\tau^-) & si \quad |\hat{s}_\psi| < \varepsilon \end{cases}$$

$$\hat{z}_1 = z_1 + n_1$$
$$\hat{s}_\psi = s_\psi + f_3$$
$$|n_1| \leq \left(\|\mathbf{z}\| + \sqrt{2}\left(1 + k_1\right)\right) n_{\max}$$
$$f_3 = k_3 k_2 k_1 p_{\max} + \left(1 + k_3 + k_2\left(1 + k_3|\dot{z}_2|\right)\left(\|\mathbf{z}\| + \sqrt{2}\left(1 + k_1\right)\right) + \right.$$
$$\left. + k_1|F_r| + 2\sqrt{2}\left|1 - d_u k_1\right|\sqrt{e_{vx}^2 + e_{vy}^2} + |r| + n_{\max}\right) n_{\max}$$

- **Position** $V = \frac{z_1^2 + z_2^2}{2}$

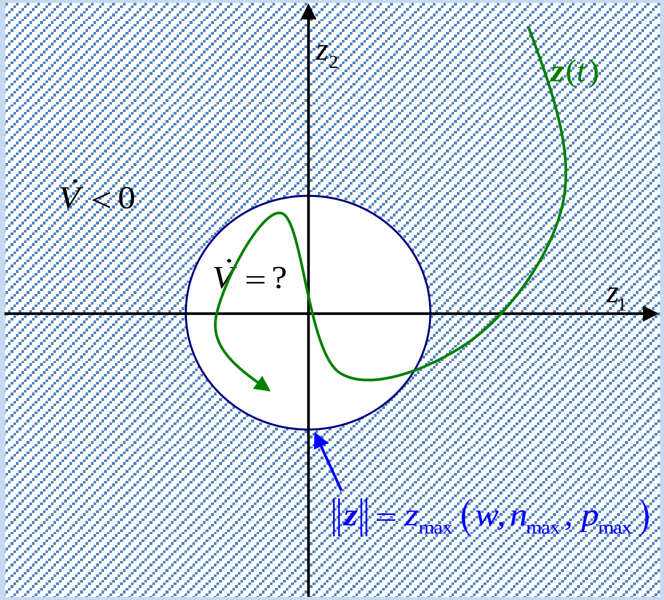
$|w| \le |\psi - \psi_c|$

$$\dot{V} = k_1 z_1 \left(F - F_r \cos(\psi - \psi_r) + p_1\right) - k_1 z_2 \left(|F_r| \sin\left(k_2 \tanh(z_2)\right) - F_r w - p_2\right) + (1 - d_u k_1)\left(s_1 e_{vx} + s_2 e_{vy}\right)$$

- Two cases:
  - A) $|z_1| > n_1 + \varepsilon$ $\quad \dot{V} \le -k_1 \|\mathbf{z}\| \left(g\left(\|\mathbf{z}\|\right) - \left(\delta\varepsilon_1 + |F_r||w| + p_{\max}\right)\right)$
  - B) $|z_1| < n_1 + \varepsilon$ $\quad \dot{V} \le -k_1 \left(|z_2| \left(|F_r| \sin\left(k_2 \tanh(|z_2|)\right) - \varepsilon_5 - \left(|F_r||w| + p_{\max}\right)\right) - \varepsilon_6\right)$
- Both cases *V* decreases outside a bounded set

$$\dot{e}_x = \frac{1}{k_1}\left(s_x - e_x\right)$$

$$\dot{e}_y = \frac{1}{k_1}\left(s_y - e_y\right)$$



$$|w| \le w_m \rightarrow \|\mathbf{e}_p(t)\| \le \beta_3\left(\|\mathbf{e}_p(t_0)\|, t - t_0\right) + \gamma_3\left(w_m + n_{\max} + p_{\max}\right)$$

- Orientation is stable for small Z
  - $||\boldsymbol{z}||<z_m$ That can be chosen arbitrary big
  - Error and noise effect is bounded

$$|f_3| \leq \gamma_6 \left(c_7 + z_m + p_{\max}\right) n_{\max} + k_3 k_2 k_1 p_{\max}$$

- Lyapunov function $V_3 = \frac{s_\psi^2}{2}$
  - $|s_y| \geq f_3 + e$ → $\dot{V}_3 \leq -\frac{\delta k_3}{4}|s_\psi| = -\frac{\delta k_3}{4}\sqrt{2V_3}$ *V_3 converge in finite time*

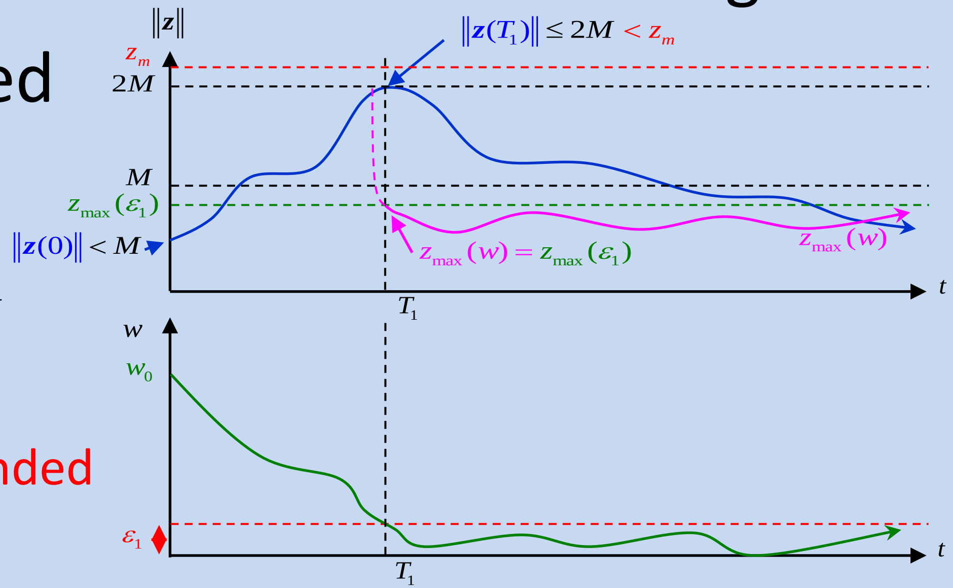
$$|\mathbf{z}| \leq z_m \rightarrow |s_\psi(t)| \leq \max\left(|s_\psi(0)| - at\ , \gamma\left(b + z_m + p_{\max}\right) n_{\max} + cp_{\max} + \varepsilon\right)$$

  - *e_ψ is ISS respect to s* $\dot{e}_\psi = \frac{s_\psi - e_\psi}{k_3}$

$$\|\mathbf{z}(t)\| \leq z_m \rightarrow |e_\psi(t)| \leq Ce^{-\frac{t}{k_3}} + \gamma_2\left(b + z_m + p_{\max}\right) n_{\max} + cp_{\max} + \varepsilon$$

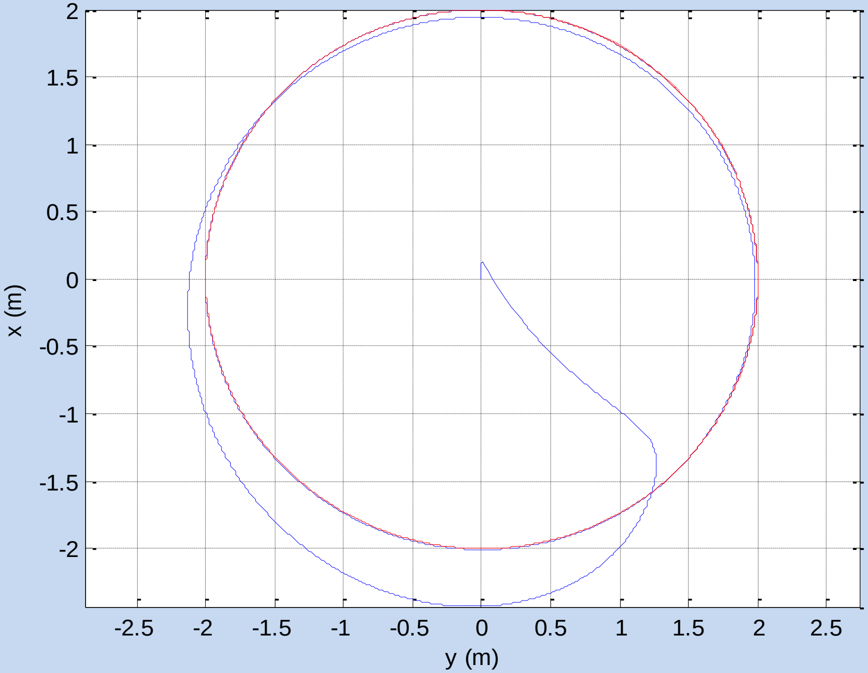
- If ***z*** is initially bounded by *M*
  - Initially z_m is not defined
    - ***z*** could increase (at a bounded rate) until w is small enough this will happen in finite time T_1
  - If $\boldsymbol{z}(t_1)$ <2*M* and z_{maz}<*M* then *w* could not diverge
  - ***z*** will be finally bounded
- Final condition

$$\sqrt{2}\tanh^{-1}\left(\frac{4\gamma_2(4M)}{\delta}\sin^{-1}3\left(2\sqrt{2}Ce^{-\frac{M}{ak_3}}\right)\right)<M$$

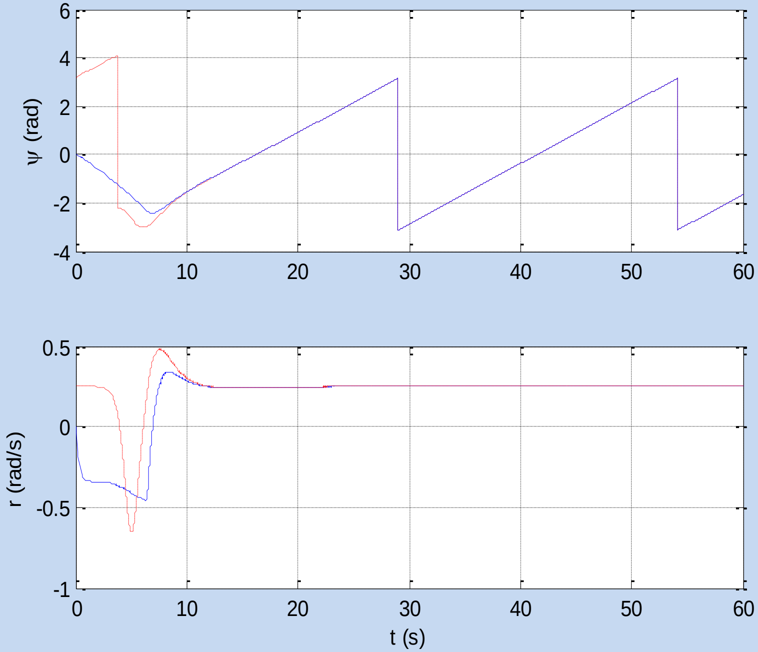


System is globally exponentially bounded if noise and disturbances are small and||***x***(0)||<*M*

- Simulated circle without noise and disturbances

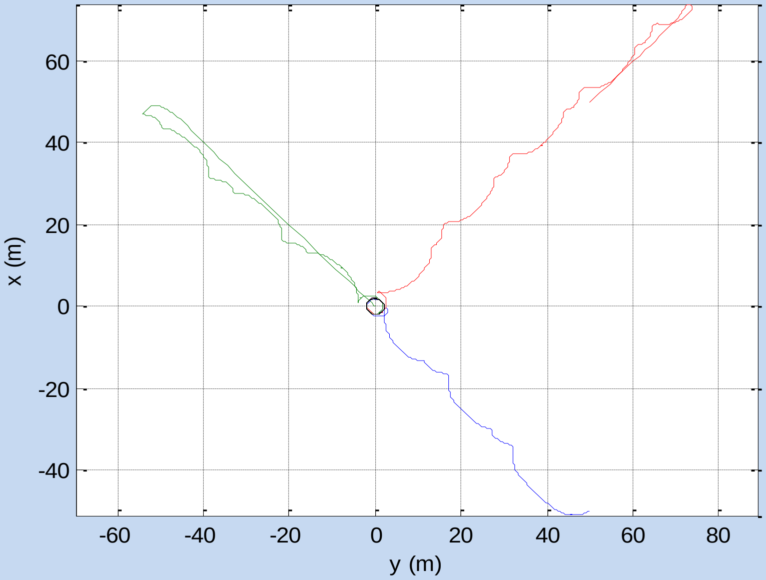


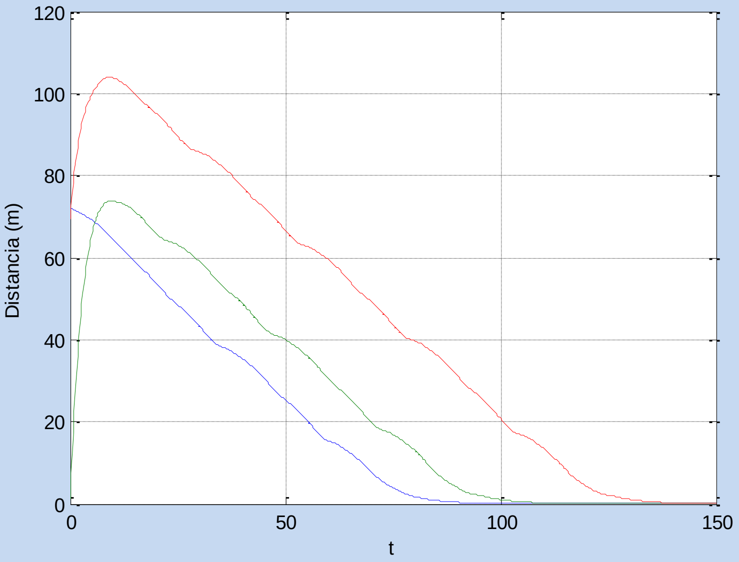
Zero initial conditions



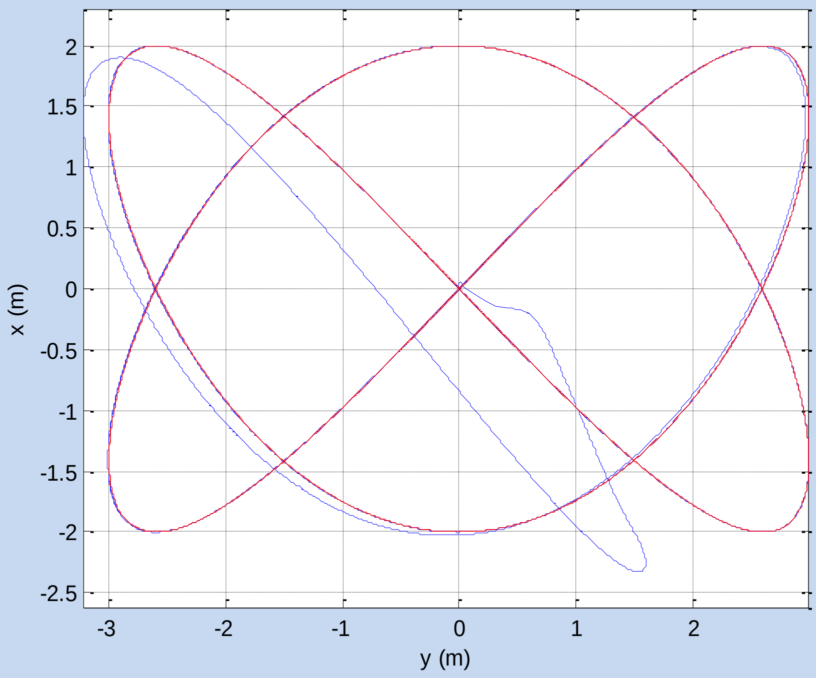
Orientation converges to ψ_c

- System converges to trajectory for very far initial conditions
  - This suggest that system is GAS

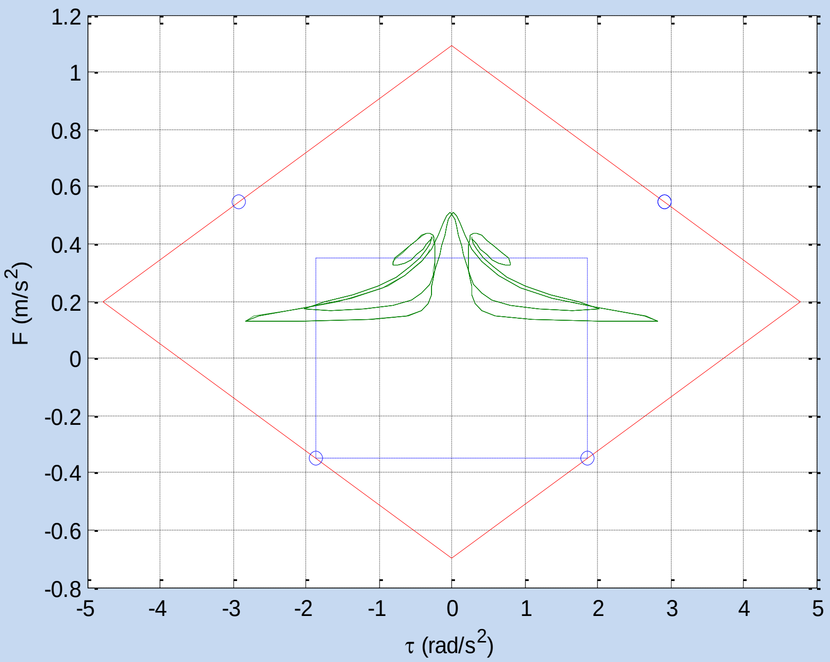




- Domination condition is conservative

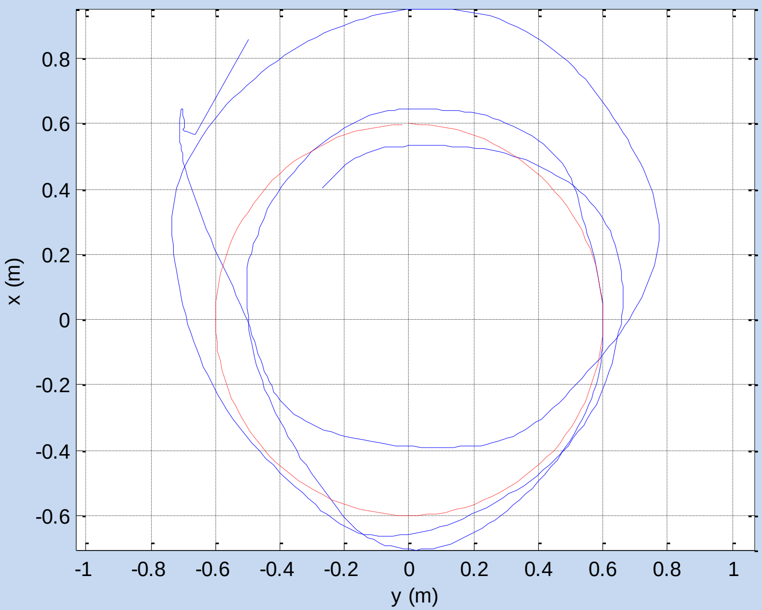


Trajectory is tracked properly

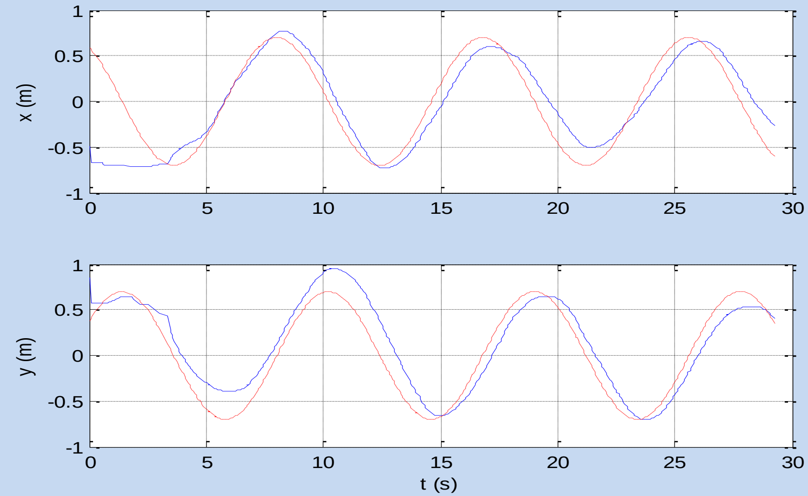


Force and torque are not dominable

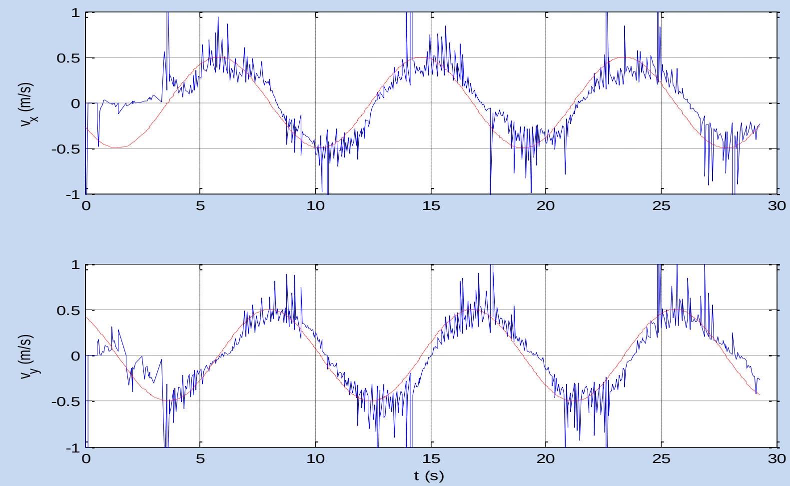
- Experimental results



Trajectory converge to a circular path

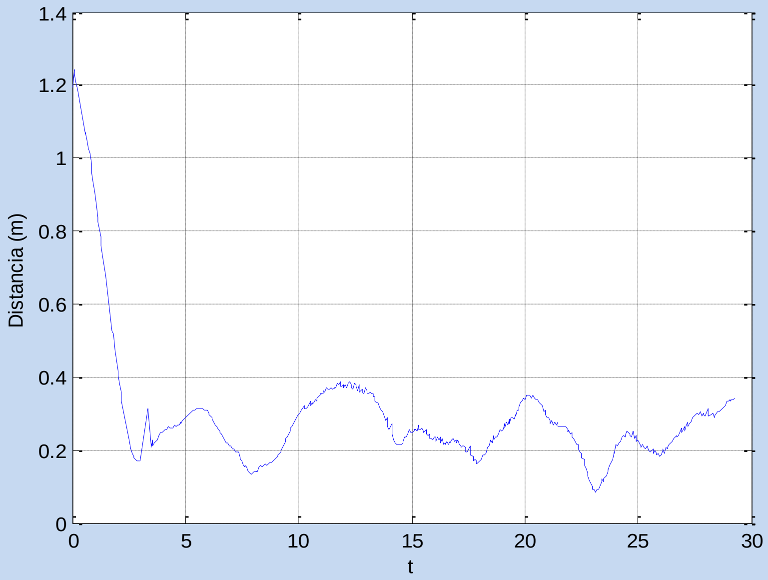


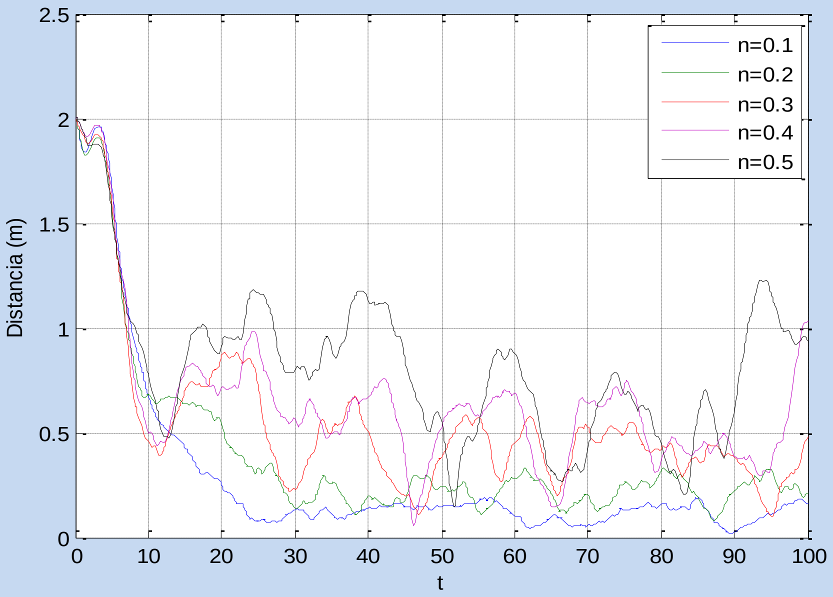
Tracking error is bounded



Velocity estimation is noisy

- Experimental VS simulation





Experiments agree with simulation with a level of noise n_{max}=0.2

Videos

Low speed tracking

High speed tracking